



# Control strategies

We need to be in control of things



**TECHNICAL  
UNIVERSITY**  
OF CLUJ-NAPOCA  
ROMANIA

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# Agenda

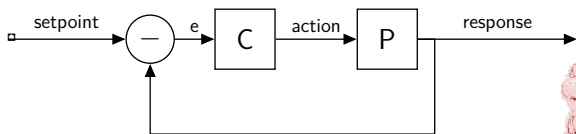
- Control theory background
- Actuator dynamics
- Independent joint control
- Computed torque control
- Force control



# Control theory

## Feedback loops

The robot is a process, and if we want to accomplish some tasks, we need to be able to control its various aspects.



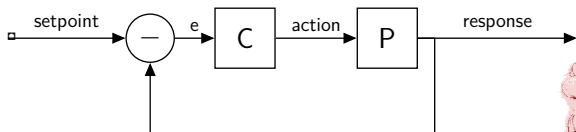
*A simple process with feedback*



# Control theory

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*A simple process with feedback*

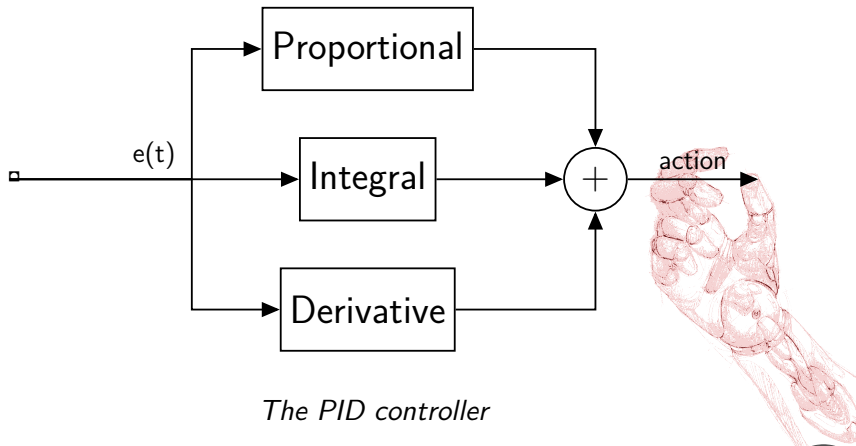
What is the setpoint? What is the response? What is the action?



# Control theory

## PID controller

One of the most basic, robust and used controller of all times!

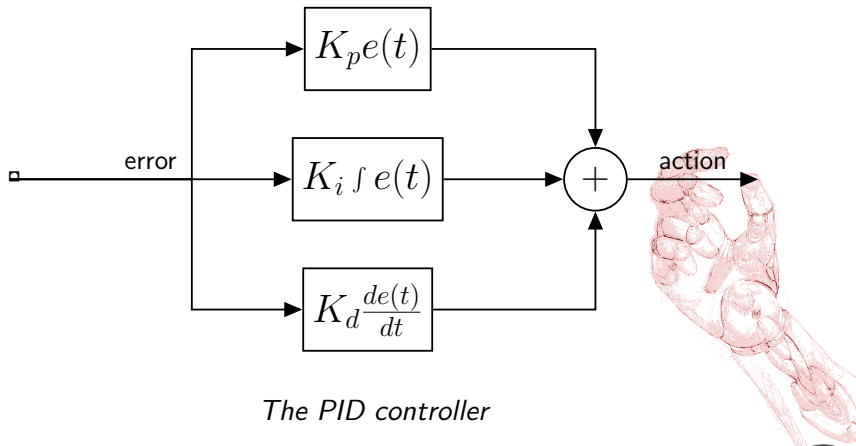


*The PID controller*

# Control theory

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# Control theory

## PID controller

How could we use such a controller for an articulated robot?



# Control theory

## PID controller

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- We could define the joint coordinates as input and end-effector pose as our output.





# Control theory

## PID controller

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- We could solve the inverse kinematics/velocities and control each joint individually.



# Control theory

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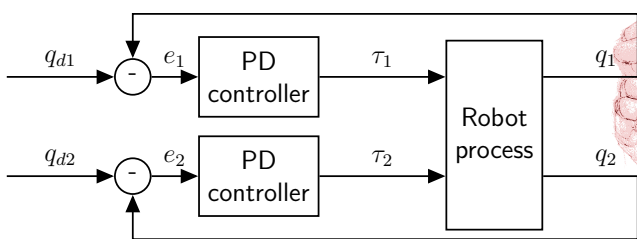
What are the pros/cons of each solution?



# Robotic controllers

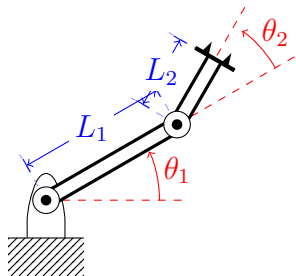
## Independent joint control

With this control strategy, we control each joint individually. If we are controlling e.g. position, then we need to solve the inverse kinematics to define the joint coordinates. These are then used as our setpoints.



# Robotic controllers

## Independent joint control

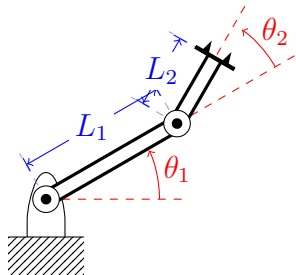


The independent joint control is considering that each joint moves independently and can therefore be controlled separately. Under which cases is this assumption correct?



# Robotic controllers

## Independent joint control



The independent joint control is considering that each joint moves independently and can therefore be controlled separately. Under which cases is this assumption correct?

The effects of inertia need to be 'small'!



# Robotic controllers

## Independent joint control

The independent joint control can take us rather far as long as:

- The motions performed are slow.
- If this is the case, then each controller can deal with the other joints motion as disturbances.
- We tune each controller diligently.



# Independent joint control

## Tuning the parameters

To tune the PID parameters analytically, we need to write a transfer function for each joint coordinate. To do that, we need the equation of motion for each joint.

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau$$



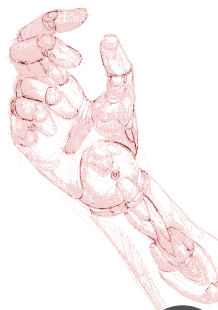
# Independent joint control

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Can we 'decouple' the joint coordinates?





# Independent joint control

## Tuning the parameters

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$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau$$

Can we 'decouple' the joint coordinates?  
Why not?



# Independent joint control

## Tuning the parameters

If we want to decouple the joint coordinates, we need for each joint to consider the effects from the motion of the other joints as disturbances.

$$d_{ii}\ddot{q}_i + c_{ii}\dot{q}_i = \tau - w$$



# Independent joint control

## Tuning the parameters

If we want to decouple the joint coordinates, we need for each joint to consider the effects from the motion of the other joints as disturbances.

$$d_{ii}\ddot{q}_i + c_{ii}\dot{q}_i = \tau - w$$

Where the term  $w$  contains all the off diagonal elements from matrices  $D$  and  $C$ , and the gravity terms  $G$ .



# Independent joint control

## Tunning the parameters

If we consider a PD controller, then the input signal becomes:

$$\tau_i = K_{Di}\dot{e}_i + K_{Pi}e_i$$

And the equation of motion becomes:

$$d_{ii}\ddot{q}_i + (c_{ii} + K_{Di})\dot{q}_i + K_{Pi}q_i = K_{Pi}q_{di} - w$$

If we consider:

$$e_i = q_{di} - q_i, \dot{e}_i = \dot{q}_{di} - \dot{q}_i, \dot{q}_{di} = 0$$



# Independent joint control

## Tuning the parameters

$$d_{ii}\ddot{q}_i + (c_{ii} + K_{Di})\dot{q}_i + K_{Pi}q_i = K_{Pi}q_{di} - w$$

This represents a second-order system, which you should know how to calculate PD parameters for a stable fast response.



# Independent joint control

## Tuning the parameters

$$d_{ii}\ddot{q}_i + (c_{ii} + K_{Di})\dot{q}_i + K_{Pi}q_i = K_{Pi}q_{di} - w$$

This represents a second-order system, which you should know how to calculate PD parameters for a stable fast response.

What happens when our assumptions are not met?



# Control theory

## System linearization

Starting from the dynamic model of the robot:

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = u$$

We are seeking an input function that can convert this model into a linear closed loop system.



# Control theory

## System linearization

Starting from the dynamic model of the robot:

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = u$$

We are seeking an input function that can convert this model into a linear closed loop system.

What about this one:

$$u = D(q)a + C(q, \dot{q})\dot{q} + G(q)$$

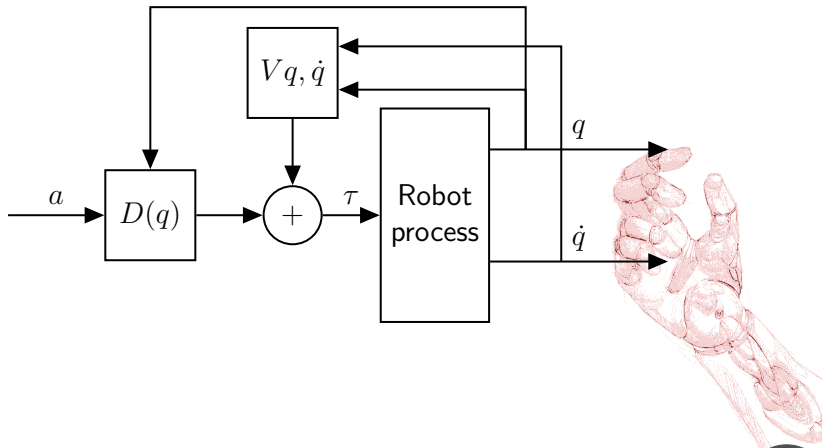
$$\ddot{q} = a$$





# Control theory

## System linearization



# Control theory

## System linearization

We can therefore 'linearize' our system, and then control it using state-space feedback for a linear system!



# Control theory

## System linearization

We can therefore 'linearize' our system, and then control it using state-space feedback for a linear system! What should our input for the new system be? (i.e.  $a$ )



# Control theory

## Computed torque control

An obvious input would be:

$$a = -K_0q - K_1\dot{q} + r$$

And the closed loop form of our system becomes:

$$\ddot{q} + K_1\dot{q} + K_0q = r$$

Where  $r$  is our reference.



# Control theory

## Computed torque control

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And the closed loop form of our system becomes:

$$\ddot{q} + K_1\dot{q} + K_0q = r$$

Where  $r$  is our reference. Choosing an  $r$  to follow the desired trajectories of  $q, \dot{q}, \ddot{q}$  like this:

$$r(t) = \ddot{q}^d(t) + K_0q^d(t) + K_1\dot{q}^d(t)$$

We end up with zero tracking error.



# Control theory

## Computed torque control

How do we calculate the desired  $q, \dot{q}, \ddot{q}$ ?





Questions?