Inverse kinematics

Cause inverse is better

TECHNICAL UNIVERSITY
OF CLUJ-NAPOCA
ROMANIA

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Agenda

- Background
- Better understanding of the DGM
- What is inverse kinematics?
- Solving
- Examples

Recap

What we saw last week?

DH Modified Parameters

We define each parameter for the length and angles from joint i until the joint $i+1$

- d_i: Joint offset (length) from joint i to joint i+1
- θ_i: Joint angle from joint i to joint i+1
- r_i: Link length from joint i to joint i+1
- α_i: Link twist (angle) from joint i to joint i+1

Recap

What we saw last week?

$$R_i^{i+1} = \left[\begin{array}{ccc|c} \cos\theta_i & -\sin\theta_i & 0 & a_i \\ \sin\theta_i\cos\alpha_i & \cos\theta_i\cos\alpha_i & -\sin\alpha_i & -d_i\sin\alpha_i \\ \sin\theta_i\sin\alpha_i & \cos\theta_i\sin\alpha_i & \cos\alpha_i & d_i\cos\alpha_i \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$R_0^n = R_0^1 * R_1^2 * \ldots * R_{n-1}^n$$

Direct Geometric Model

Better understanding

Definition

A transformation matrix that calculates the orientation and position of the robot's end effector in terms of the joint coordinates $q_1, q_2, \ldots, q_n$

$$\left[\begin{array}{ccc|c} X_X & Y_X & Z_X & P_x \\ X_Y & Y_Y & Z_Y & P_x \\ X_Z & Y_Z & Z_Z & P_x \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Direct Geometric Model

Better understanding

This is basically a function of $q = [q_1, q_2, \ldots, q_n]$ that returns the orientation and position of the end-effector

$$f(q) \mapsto P_x, P_y, P_z, R$$

Where R is the orientation defined in terms of $X_x, X_y, \ldots$

Inverse Geometric Model

Definition

It is basically the inverse of the direct geometric model

$$g(P_x, P_y, P_z, R) \mapsto q = [q_1, q_2, \ldots, q_n]$$

A function that given a specific position and orientation, returns the joint coordinates.

Inverse and Direct models

What is the difference?

Direct geometric model

I want to know where will my end-effector be, if I position each joint to a specific position

Inverse geometric model

I want to know what should the joint coordinates be in order for my end-effector to achieve a specific position

Inverse and Direct models

What is the difference?

For robotics applications, the inverse model is way more useful.

Inverse and Direct models

What is the difference?

For robotics applications, the inverse model is way more useful.

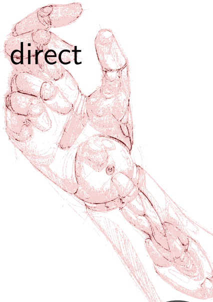
Can you understand why?

Inverse and Direct models

What is the difference?

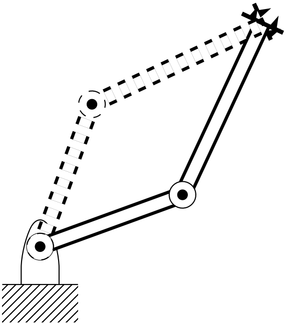
For robotics applications, the inverse model is way more useful.

But it is also most difficult to derive and we need the direct geometric model to derive it.



Inverse kinematic model

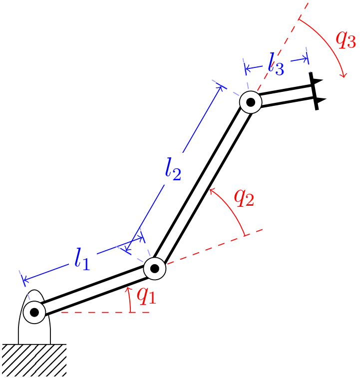
Why so difficult?



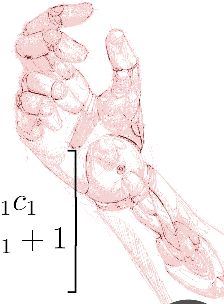
The inverse kinematic model might have more than one solution for a specific robot position

Inverse kinematic model

Why so difficult?



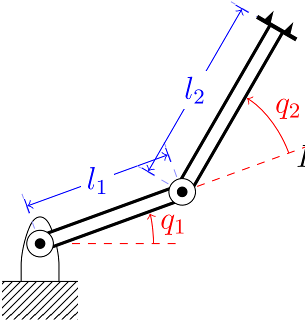
$$R_0^4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{1,2,3} & -s_{1,2,3} & l_2 c_{1,2} + l_3 c_{1,2,3} + l_1 c_1 \\ 0 & s_{1,2,3} & c_{1,2,3} & l_2 s_{1,2} + l_3 s_{1,2,3} + l_1 s_1 + 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Inverse kinematis model

Derivation

The inverse model can be difficult to solve even for simple models



$$R(q_1, q_2) = \begin{bmatrix} c_{1,2} & -s_{1,2} & 0 & l_2 c_{1,2} + l_1 c_1 \\ s_{1,2} & c_{1,2} & 0 & l_2 s_{1,2} + l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Inverse kinematics model

Derivation

$$\begin{bmatrix} c_{1,2} & -s_{1,2} & 0 & l_2 c_{1,2} + l_1 c_1 \\ s_{1,2} & c_{1,2} & 0 & l_2 s_{1,2} + l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} X_X & Y_X & Z_X & P_x \\ X_Y & Y_Y & Z_Y & P_x \\ X_Z & Y_Z & Z_Z & P_x \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Inverse kinematics model

Derivation

$$\begin{bmatrix} c_{1,2} & -s_{1,2} & 0 & l_2c_{1,2}+l_1c_1 \\ s_{1,2} & c_{1,2} & 0 & l_2s_{1,2}+l_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} X_X & Y_X & Z_X & P_x \\ X_Y & Y_Y & Z_Y & P_x \\ X_Z & Y_Z & Z_Z & P_x \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$cos(q_1+q_2) = X_x = Y_y$$

$$sin(q_1+q_2) = X_y = -Y_x$$

$$l_2cos(q_1+q_2) + l_1cosq_1 = P_x$$

$$l_2sin(q_1+q_2) + l_1sinq_1 = P_y$$

$$0 = P_z$$

Inverse kinematics model

Derivation

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$$0 = P_z$$

How much 'freedom' do we have?

Inverse kinematics model

Derivation

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$$0 = P_z$$

How much 'freedom' do we have?
How do we solve this?

Inverse kinematics model

Geometric solution

We are looking for this

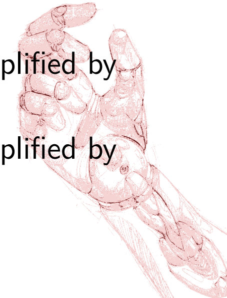
$$q_1 = f(P_x, P_y)$$
$$q_2 = g(P_x, P_y)$$

Inverse kinematics model

Analytical solutions

The idea is to try to find an equation for each joint variable q_n that only depends on the poze or on other joint variables that have already been expressed in terms of the poze.

- We equate the DGM with the general homogeneous matrix
- We identify joint variables that can be isolated
- We identify pair of joint variables that can be simplified by division
- We identify pair of joint variables that can be simplified by trigonometry



Inverse kinematics model

Analytical solutions

If not all joint variables are expressed as a function of the poze, we multiply from left(right) the inverse transformation of the first(last) joint.

$$R_0^n = R_0^1 R_1^2 \dots R_{n-1}^n = R_g$$

$$(R_0^1)^{-1} R_o^n = (R_0^1)^{-1} R_g \text{ or } R_o^n (R_{n-1}^n)^{-1} = R_g (R_{n-1}^n)^{-1}$$

And we try to isolate again

Inverse kinematics model

Examples

Inverse kinematics model

Numerical solutions

Optimisation methods, sem 2, 4th year!