# Modelling in Biomechanics

Summary

Last update: May 17, 2021

# Agenda

- Materials
- Finite Elements Analysis
- Tissues of interest
- Fluid Dynamics
- Musculoskeletal modelling
    - Muscle models
    - Forward/inverse kinematics
    - Forward/inverse dynamics

# Materials

# Strength of materials

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The study of describing the amount of load that can be exerted on a material until it deforms or fails.

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The study of describing the amount of load that can be exerted on a material until it deforms or fails.

Galileo Galilei was one of the first to develop a theory for the strength of materials *(Two new sciences, 1638)*

What concepts you remember from 'Rezistența Materialelor'?

# Strength of materials

## General overview

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**Deformation** can be either temporary or permanent.

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## Stress

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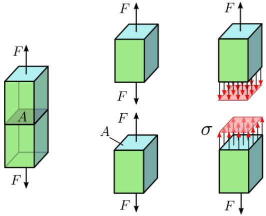
$$\sigma = \frac{F}{A}$$

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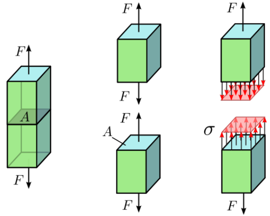


Why use stress instead of force?

# Strength of materials

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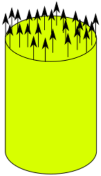

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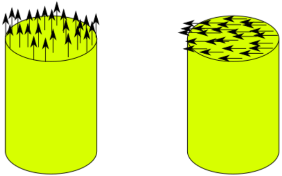

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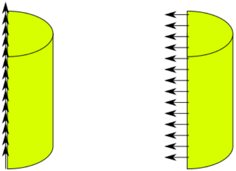

What is *normal* or *parallel*, depends what is the surface of reference

# Strength of materials

## Tensile tests

How do we quantify the properties of a material?

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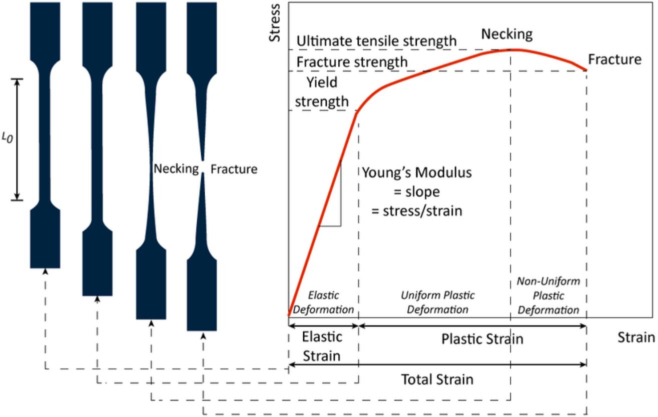

# Strength of materials

## Isotropic vs Anisotropic

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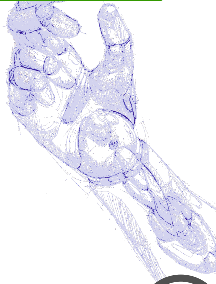
A material that has the same properties regardless of the axis of measurement

### Anisotropic material

A material that its properties differ along different axes.



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## Isotropic vs Anisotropic

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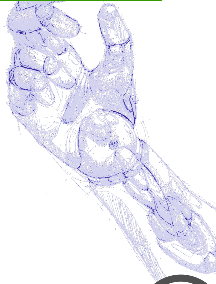
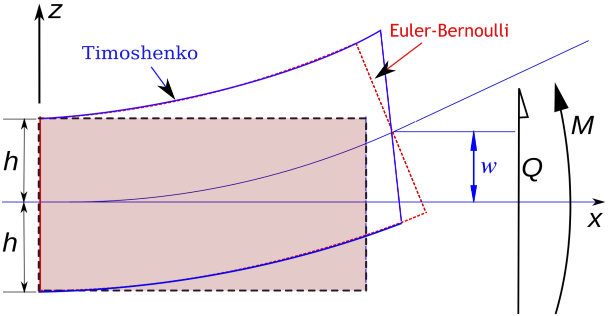
What can be the source of anisotropy?
What kind do you think biological materials are?



# Strength of materials

## Timoshenko-Ehrenfest beam theory

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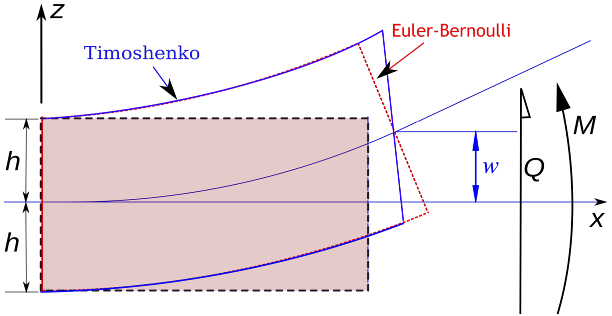


$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(EI\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}\right) = q(x)$$

Why is Euler-Bernouli wrong?

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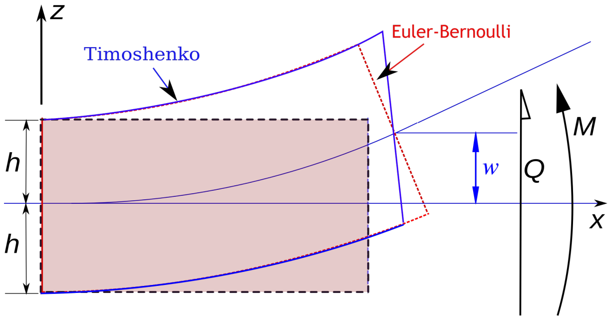


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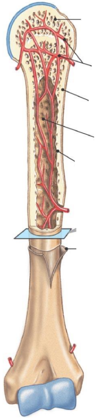

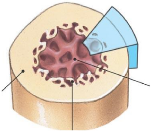

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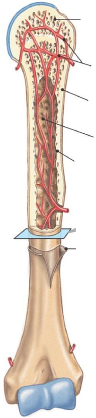

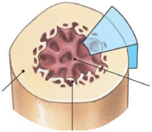

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Why not?

# Finite Elements Analysis

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A computational scheme to solve field problems. The field can be *stress*, *heat*, *pressure*, *electric*, *magnetic*, etc, etc. The principle involves dividing the body in finite pieces that can provide analytical solutions.

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Key word is **discretization**

# Finite Element Analysis

## Different levels of discretization

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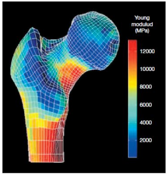


- Geometry

# Finite Element Analysis

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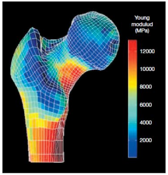


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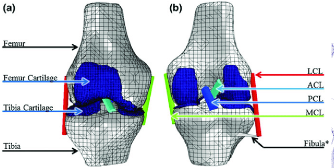
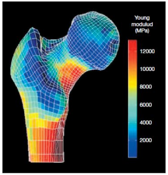
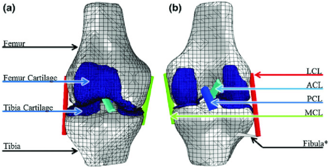


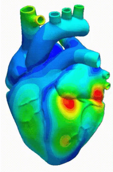
# Finite Element Analysis

## Different levels of discretization

- Geometry
- Materials
- Time



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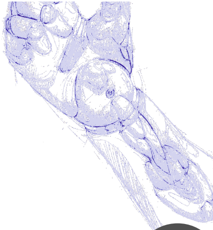

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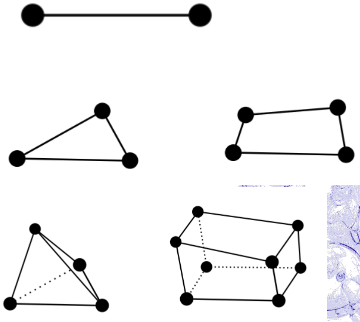

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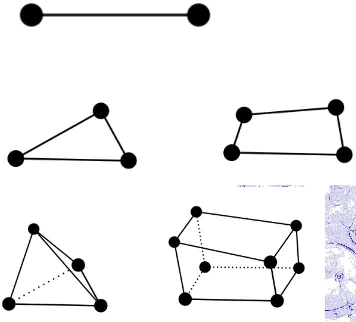

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What does it mean 1D, 2D, 3D?

# Finite Element Analysis

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A model of a spring.

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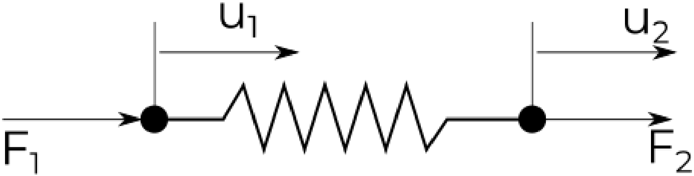

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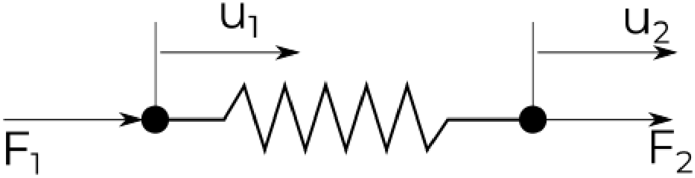


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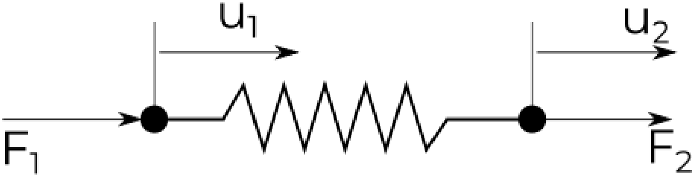


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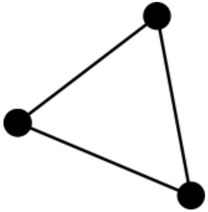
$$\begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

# Finite Element Analysis

## 2D element equations



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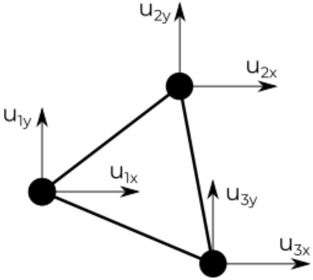

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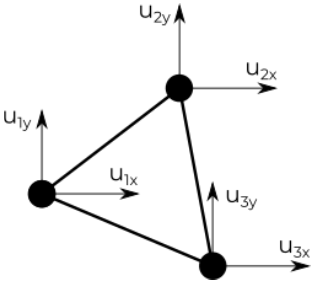


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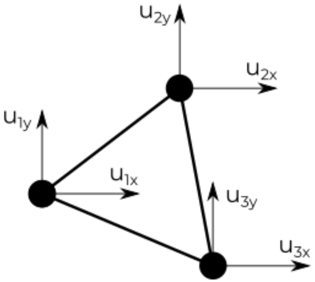


$k = tA(B^T EB)$

where:

- t: Thickness of the plate
- A: Area of the triangle
- E: Young's modulus
- B: "shape" matrix

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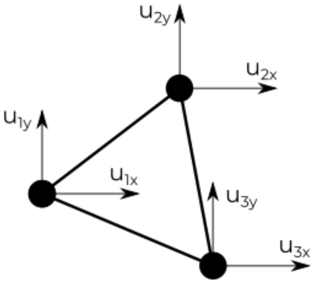


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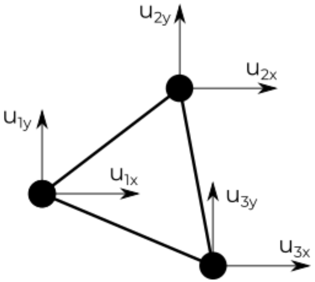


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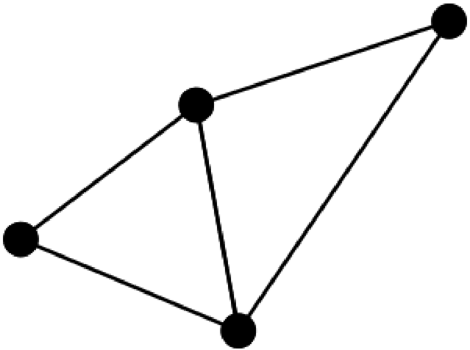
$x_{ij} = x_i - x_j$

# Finite Element Analysis

## Combining elements



How do we combine elements?

# Finite Element Analysis

## Problem construction

In any of these cases, we are trying to solve a problem of force and displacement

$$\left[K\right]\left\{u\right\} = \left\{F\right\}$$

# Finite Element Analysis

## Applications in biomechanics

Why is this useful for biomechanics?

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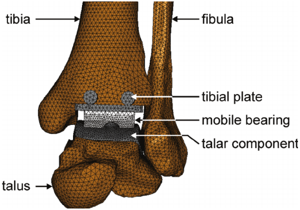


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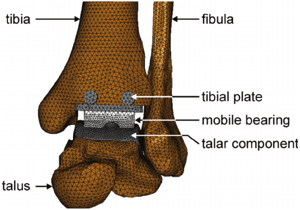


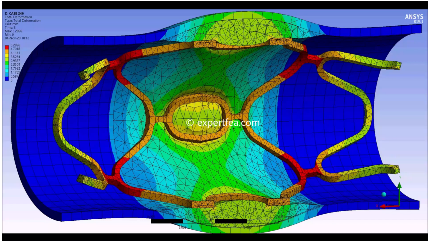


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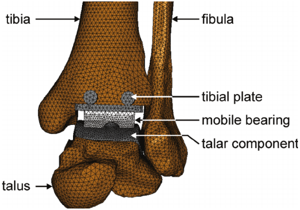
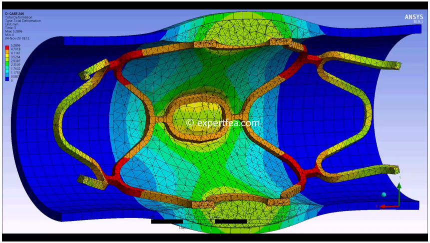
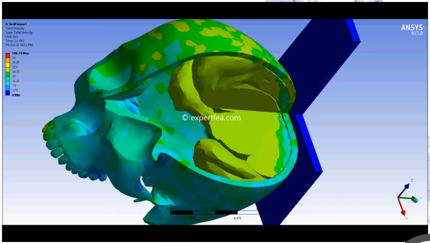


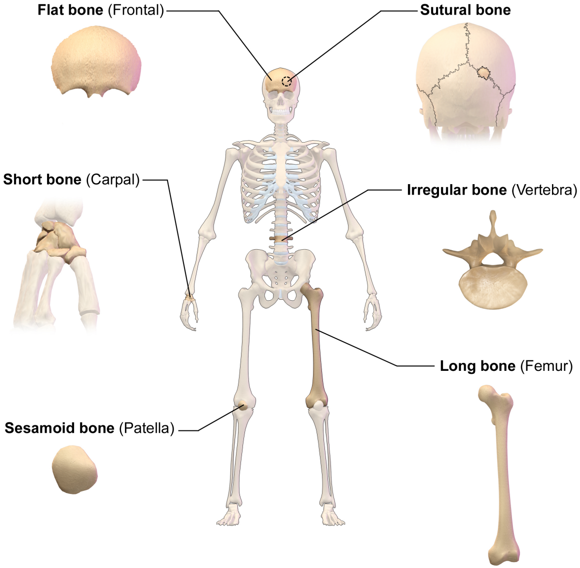



# Tissues of interest

# Bone morphology

## Types of bones




# Bone morphology

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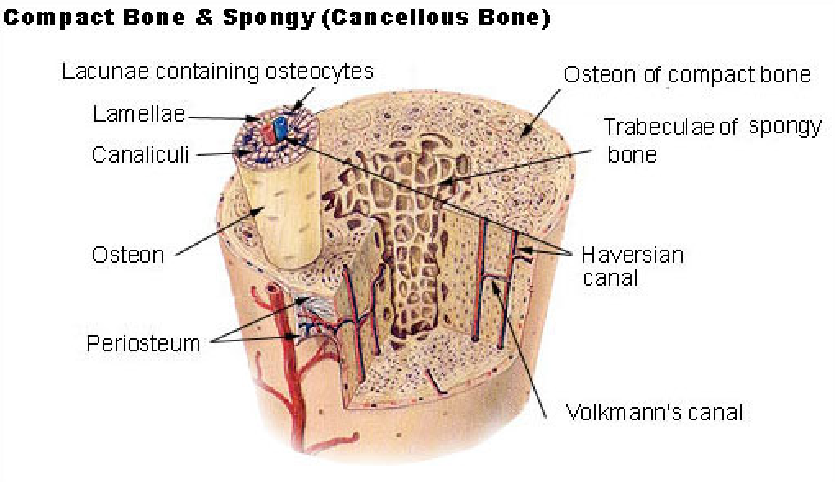
https://www.youtube.com/watch?v=0dV1Bwe2v6c

# Composition of long bones



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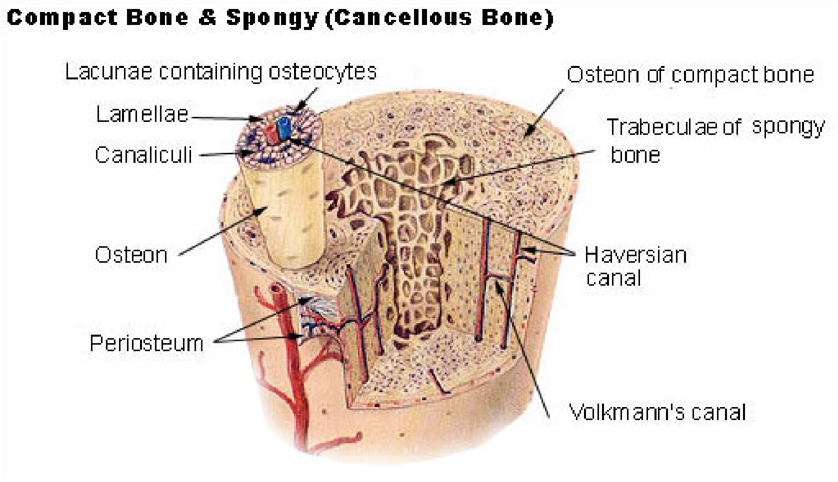


Basic structure is the osteon

# Bone properties

## Modelling

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Where $p$ is the fluid pressure, and $A$ is called the Biot coefficient

# Cartilage

## Chemical Composition

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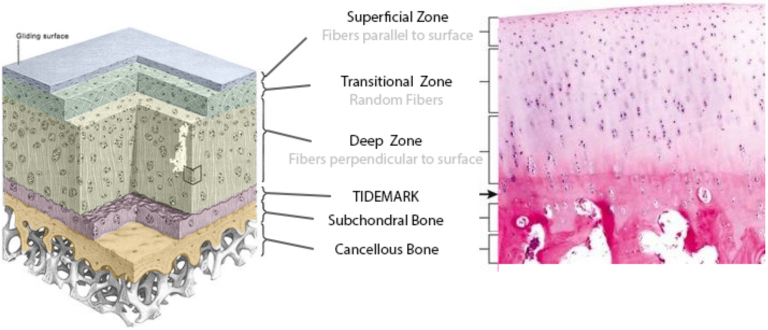

# Cartilage

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The property of a porous material that describes the ability of a fluid to flow through the material.

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**Permeability**

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This is a non-linear model

# Cartilage

## Modelling

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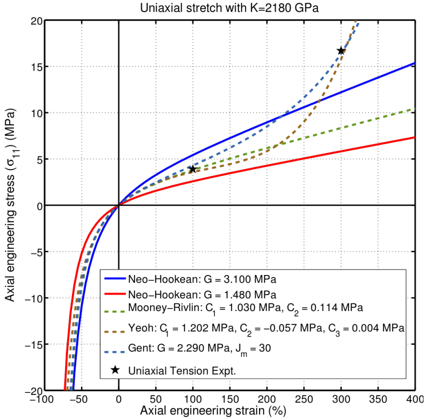


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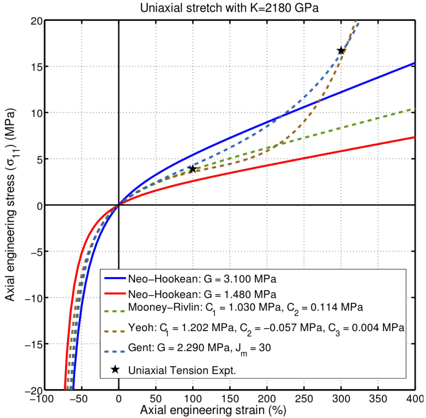



More specifically a *Mooney-Rivlin* material

# Tendons and ligaments

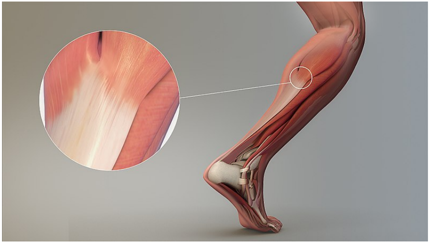
## Functionality

# Tendons and ligaments

## Functionality

### Tendons

Fibrous connective tissue connecting muscles to bones. They help translate muscle force production into bone movement.



# Tendons and ligaments

## Functionality

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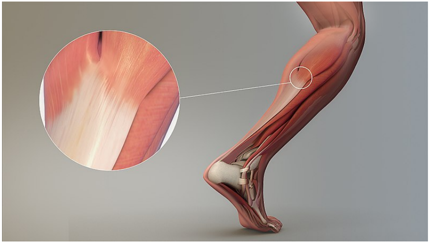
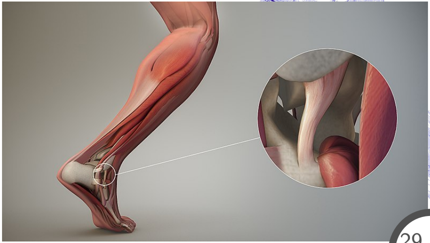
### Ligaments

Fibrous connective tissue connecting bones to bones. They help keep bones together, restricting some degrees of freedom in articulations.





# Tendons and ligaments

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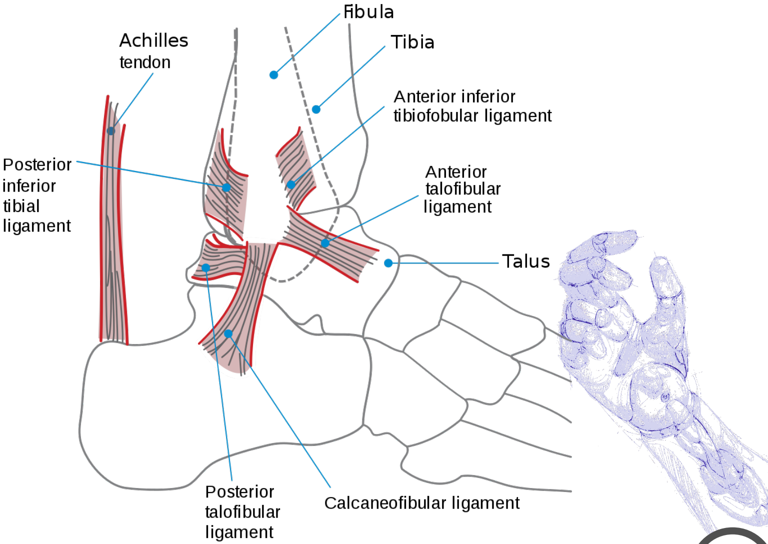

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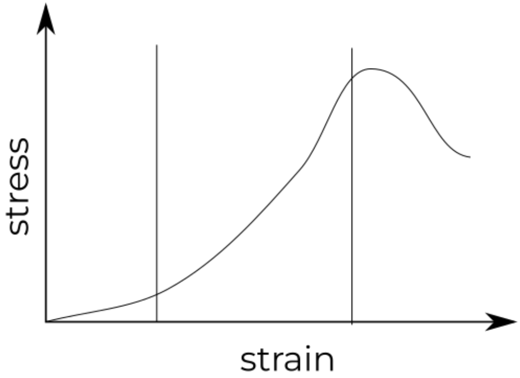



# Tendons and ligaments

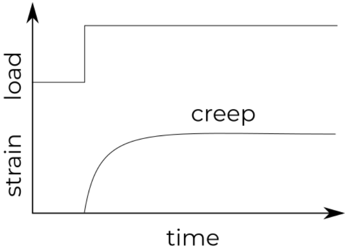
## Modelling

Viscous effects

# Tendons and ligaments

## Modelling

### Viscous effects




# Tendons and ligaments

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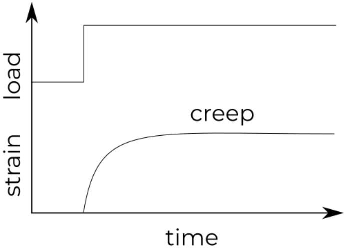


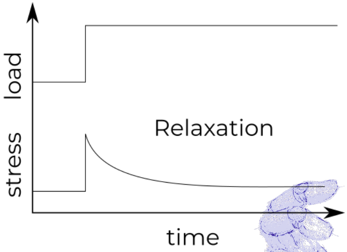

# Tendons and ligaments

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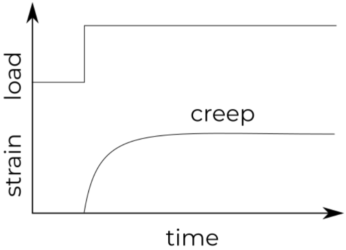
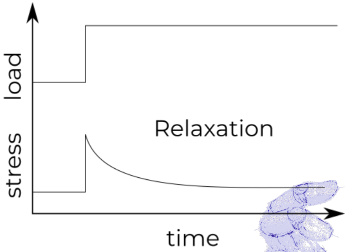
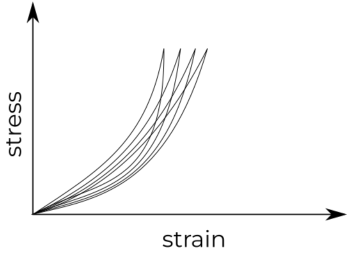
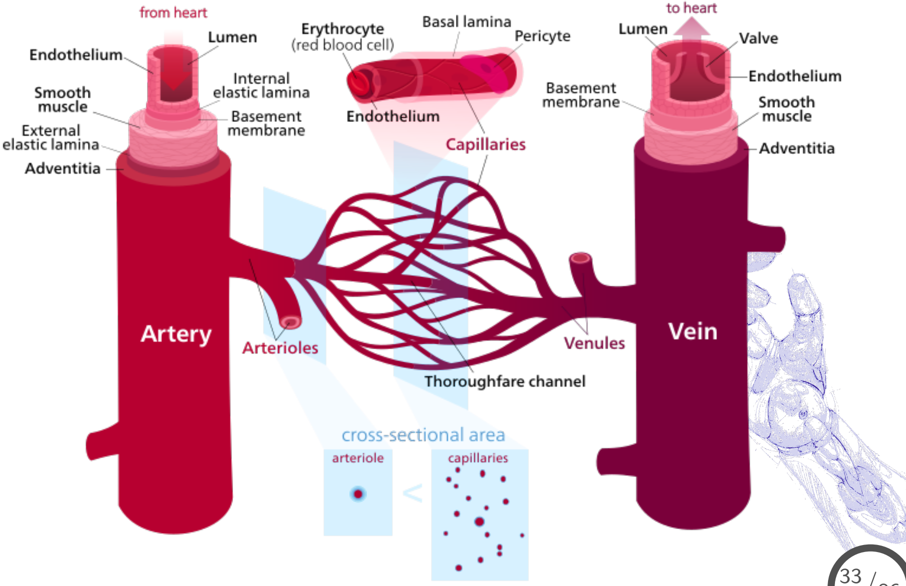


# Cardiovascular system

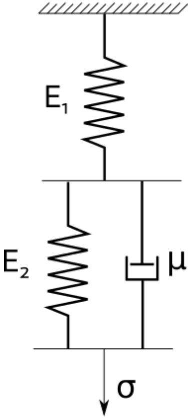
## Description



# Blood vessels

## Mechanical properties

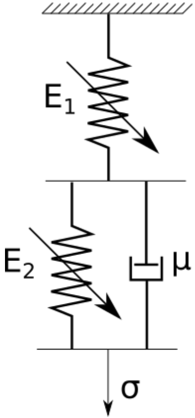
Visco



# Blood vessels

## Mechanical properties

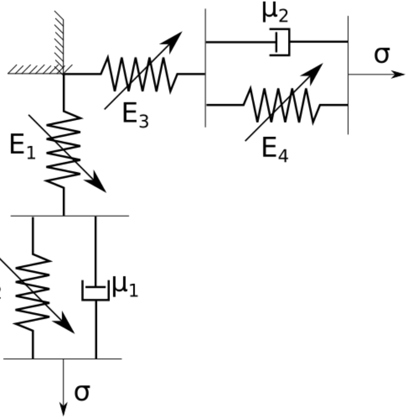
Viscohyperelastic




# Blood vessels

## Mechanical properties

Viscohyperelastic, anisotropic



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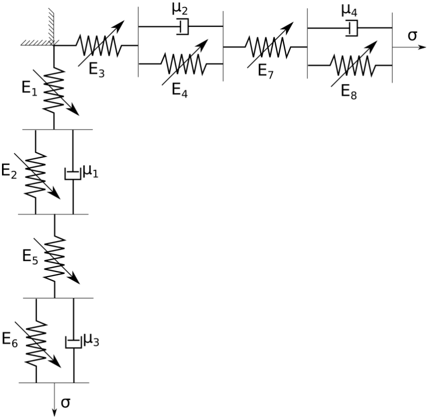
Viscohyperelastic, anisotropic, composite.

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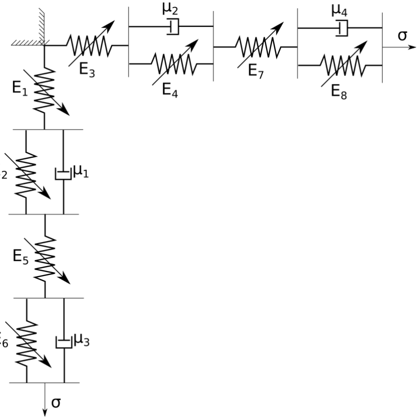
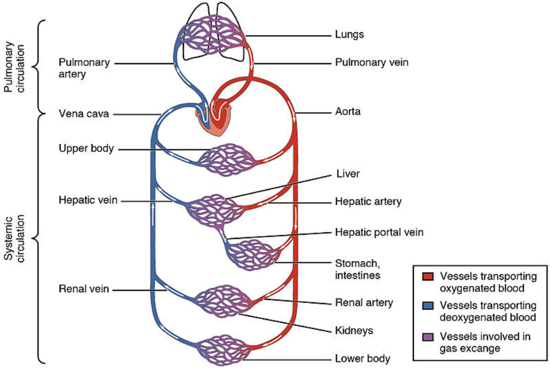



And it exhibits residual stresses!

# Fluid Dynamics

# Fluid systems

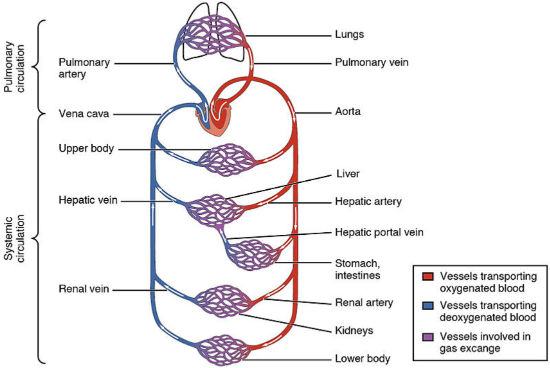
## Cardiovascular system




From *Anatomy & Physiology, Connexions Web site*

# Fluid systems

## Cardiovascular system

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- Pressure in vessels

# Fluid systems

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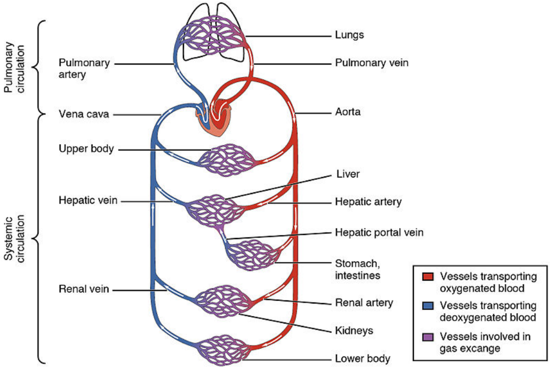


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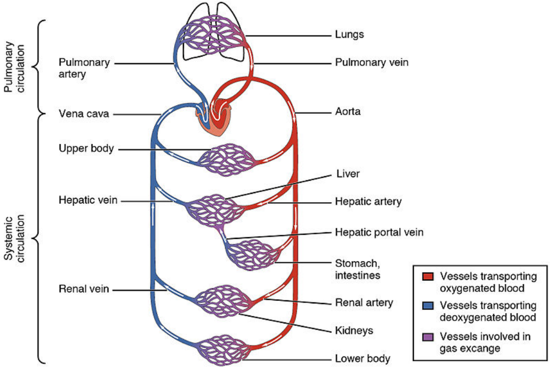
# Fluid systems

## Cardiovascular system




From *Anatomy & Physiology, Connexions Web site*

- Pressure in vessels
- Blood flow rate
- Turbulence

## Incomplressible flow equation

Daniel Bernoulli 1700-1782

# Fluid mechanics

## Basic principles

Incomplressible flow equation

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$$\frac{u^2}{2} + gz + \frac{p}{\rho} = constant$$

Incomplressible flow equation

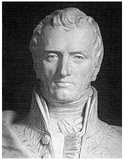
Daniel Bernoulli 1700-1782

$$\frac{u^2}{2} + gz + \frac{p}{\rho} = constant$$

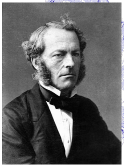
$u$: fluid flow speed
$g$: gravitational acceleration
$z$: elevation
$p$: pressure
$\rho$: fluid density

# Fluid mechanics

## Navier-Stokes equations



Claude-Lois Navier

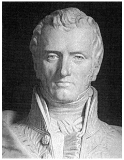


Sir George Stokes

# Fluid mechanics

## Navier-Stokes equations

$$\nabla \vec{u} = 0$$



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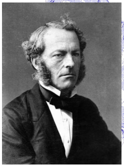

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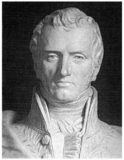

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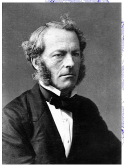

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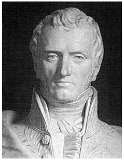

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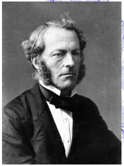

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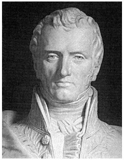

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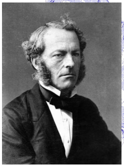

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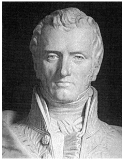
(Newton's second law F=ma)

We don't understand these fully!

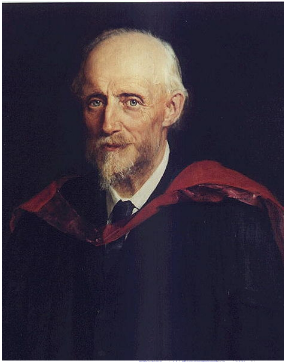


Claude-Lois Navier

Sir George Stokes

# Fluid mechanics

## Reynolds number

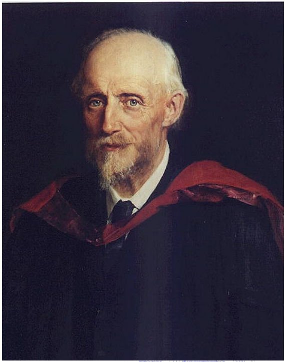


Osborne Reynolds 1842-1912

# Fluid mechanics

## Reynolds number

$$Re = \frac{\rho u L}{\mu}$$

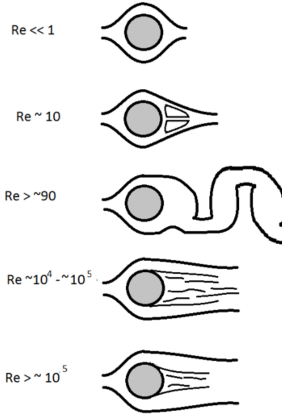


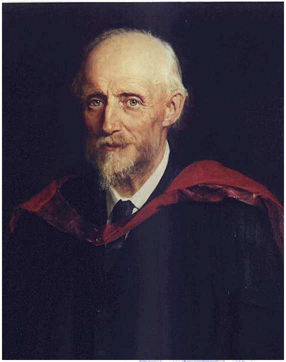
Osborne Reynolds 1842-1912

# Fluid mechanics

## Reynolds number

$$Re = \frac{\rho u L}{\mu} = \frac{F_{inertia}}{F_{viscous}}$$



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# Fluid mechanics

## Reynolds number

$$\rho \frac{\partial u}{\partial t} + \rho \vec{u} \frac{D\vec{u}}{Dt} = -\nabla p + \mu \nabla^2 \vec{u} + \rho F$$

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# Fluid mechanics

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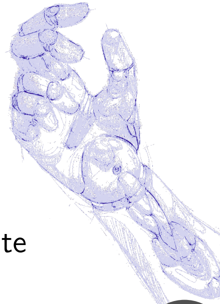
For Re $\gg$1:

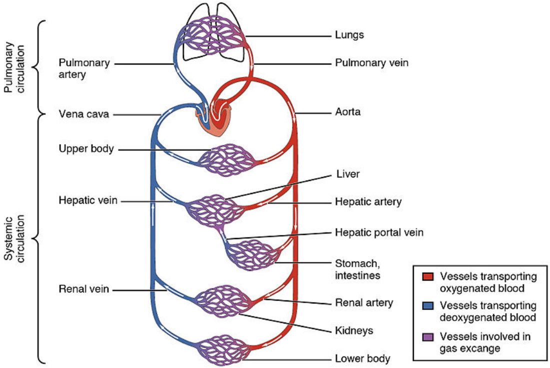
$$\rho\frac{\partial u}{\partial t} + \rho\vec{u}\frac{D\vec{u}}{Dt} = -\nabla p$$

Either of these are much simpler to compute



# Fluid mechanics

## Reynolds number in cardiovascular system




From *Anatomy & Physiology, Connexions Web site*

# Fluid mechanics

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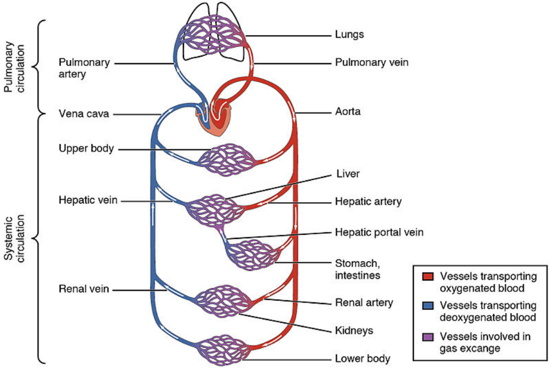


From *Anatomy & Physiology, Connexions Web site*

- Ascending Aorta: 4500

# Fluid mechanics

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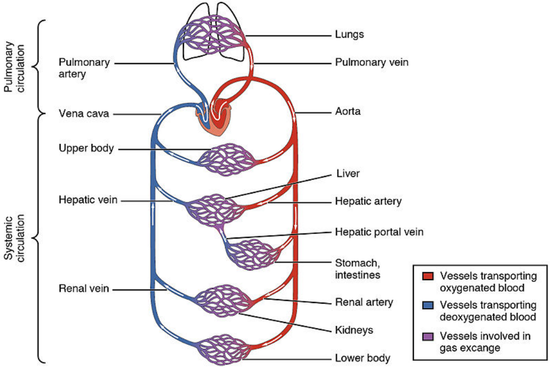
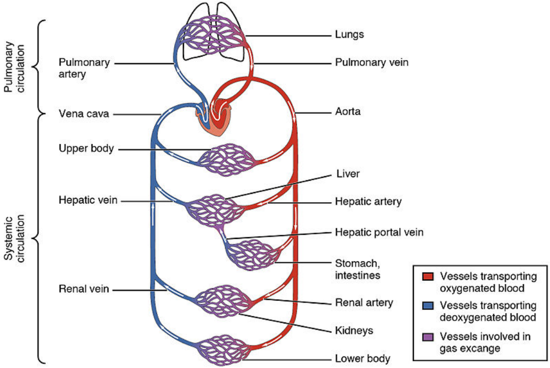



From *Anatomy & Physiology, Connexions Web site*

- Ascending Aorta: 4500
- Descending Aorta: 3400

# Fluid mechanics

## Reynolds number in cardiovascular system

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# Fluid mechanics

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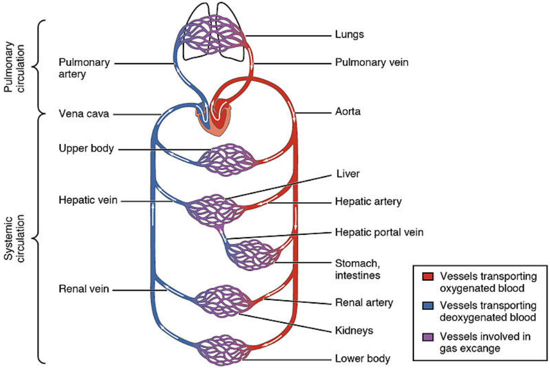


From *Anatomy & Physiology, Connexions Web site*

- Ascending Aorta: 4500
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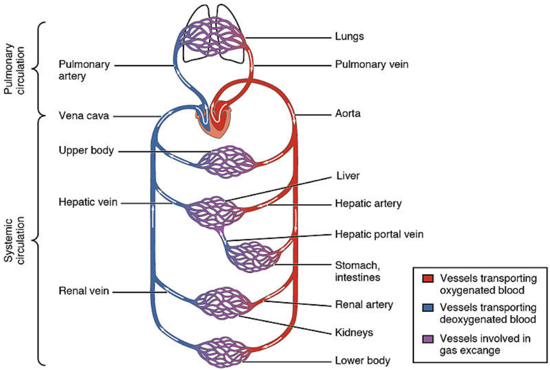
- Abdominal Aorta: 1250
- Femoral artery: 1000

# Fluid mechanics

## Reynolds number in cardiovascular system

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- Ascending Aorta: 4500
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# Fluid mechanics

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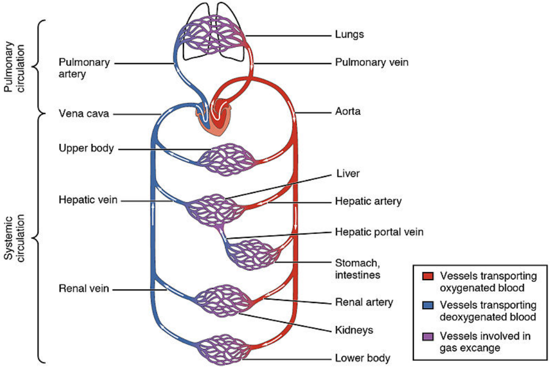


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- Capilaries: 0.001

# Fluid mechanics

## Hagen-Poiseuille flow

Considering steady flow:

$$\Delta P = \frac{8\pi\mu L Q}{A^2}$$

# Fluid mechanics

## Hagen-Poiseuille flow

Considering steady flow:

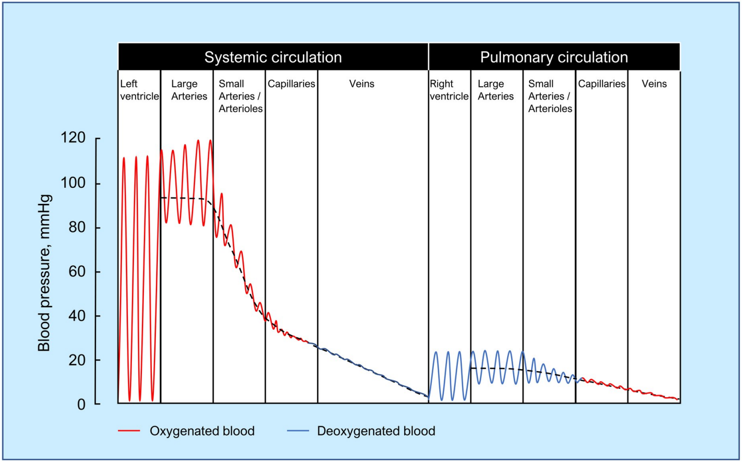
$$\Delta P = \frac{8\pi\mu L Q}{A^2}$$

$\Delta P$: Pressure drop
$\mu$: Viscocity
$L$: Length
$Q$: Flow rate
$A$: Crossectional area

# Fluid mechanics

## Pressure drop

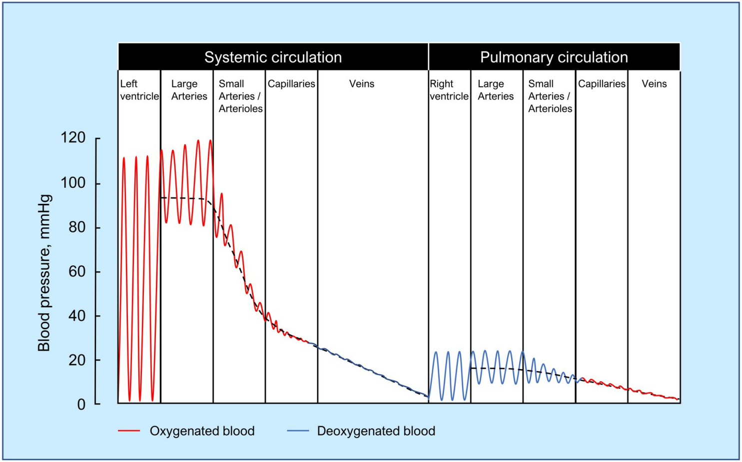
$$\Delta P = \frac{8\pi\mu L Q}{A^2}$$



# Fluid mechanics

## Pressure drop

$$\Delta P = \frac{8\pi\mu L Q}{A^2}$$ , A way to calculate pressure along the cardiovascular system



# Blood characteristics

## Fahraeus-Lindqvist effect

Blood viscosity drops at very small diameters (capilaries)

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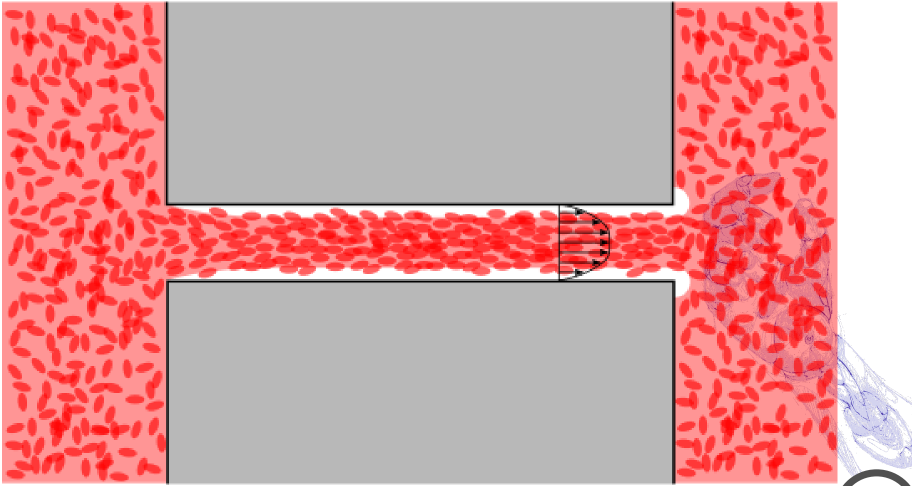
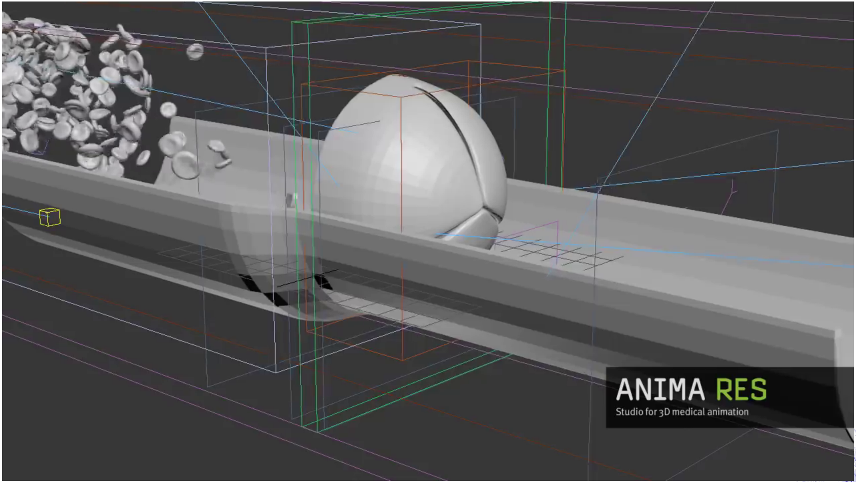
## Fahraeus-Lindqvist effect

Blood viscosity drops at very small diameters (capilaries)



# Fluid mechanics

## Flow in elastic walls




Anima RES youtube channel

# Fluid biomechanics

## How do we combine everything together?

A lot of complex phenomena, bring the models to its limits.

# Fluid biomechanics

## How do we combine everything together?

A lot of complex phenomena, bring the models to its limits.



# Musculoskeletal modelling

# Muskuloskeletal modeling

## Human anatomy

What does the word 'musculoskeletal' mean to you?

# Muskuloskeletal modeling

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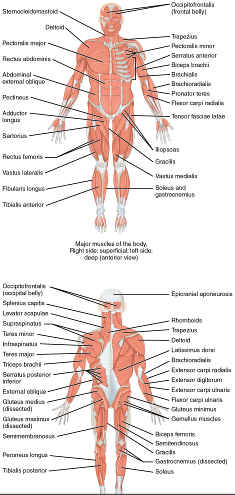


Major muscles of the body. Right side: superficial; left side: deep (anterior view)

# Articulated joints

## Types of joints

Characterization based on degrees of freedom

# Articulated joints

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Characterization based on **degrees of freedom**

# Articulated joints

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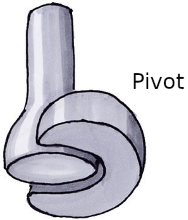
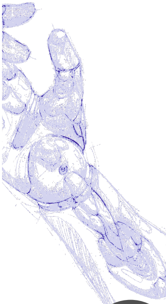
Characterization based on **degrees of freedom**




By Produnis



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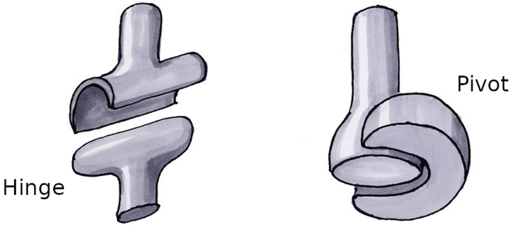


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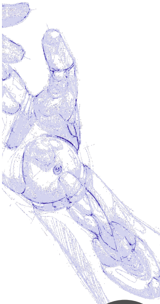

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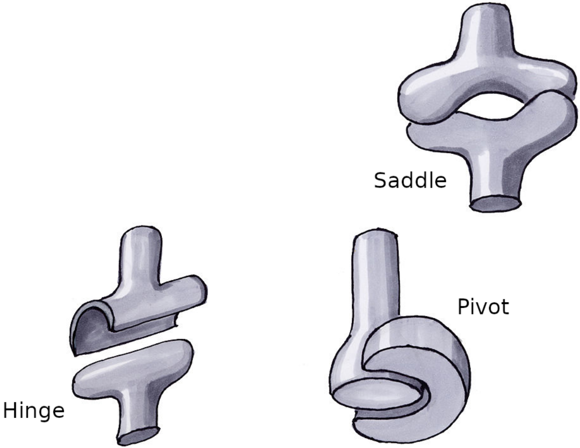


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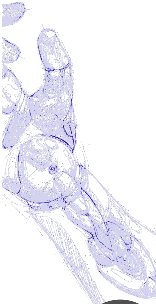

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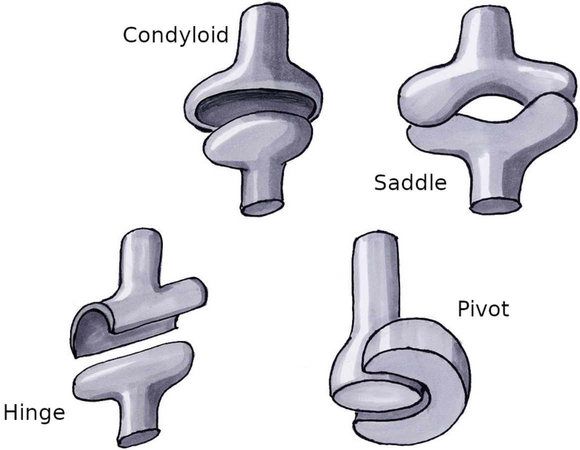


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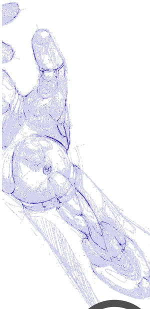

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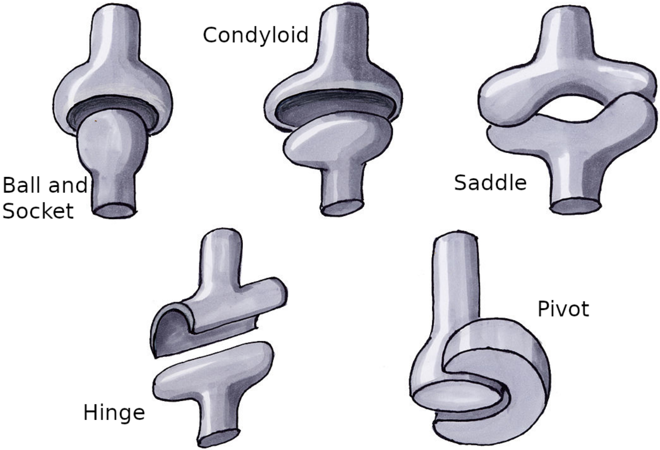


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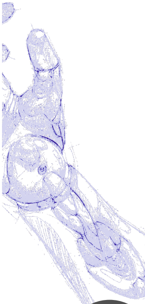


# Musculoskeletal modelling

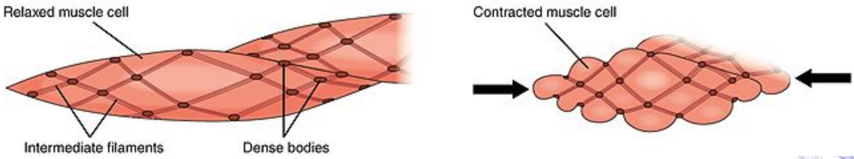
## Movement production

Neural Command

# Musculoskeletal modelling

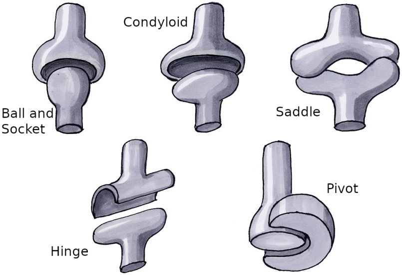
## Movement production

**Neural Command** → **Musculotendon Dynamics**



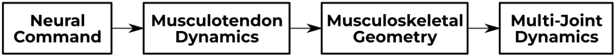
# Musculoskeletal modelling

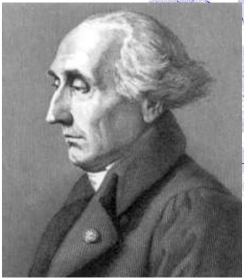
## Movement production

# Musculoskeletal modelling

## Movement production



# Musculoskeletal modelling

## Movement production

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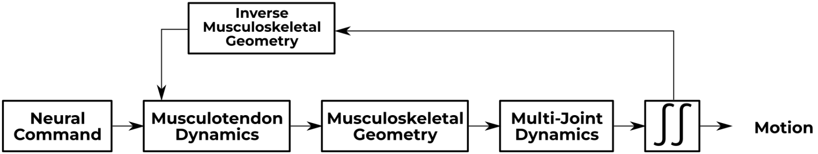
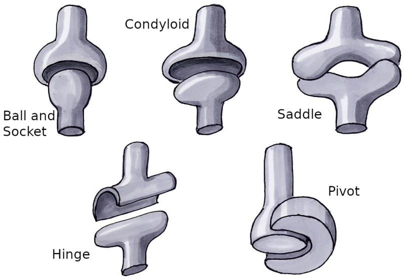
## Movement production




# Musculoskeletal modelling

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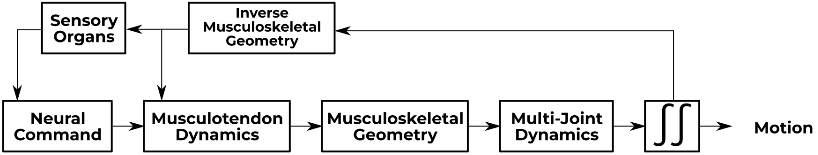


# Muscle modelling

# Musculoskeletal modelling

## Muscles

There are three types of muscles:

# Musculoskeletal modelling

## Muscles

There are three types of muscles:

- Skeletal

# Musculoskeletal modelling

## Muscles

There are three types of muscles:

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- Smooth

# Musculoskeletal modelling

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There are three types of muscles:

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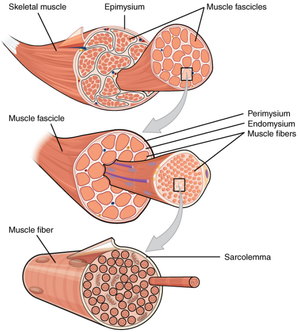
# Musculoskeletal modelling

## Muscles

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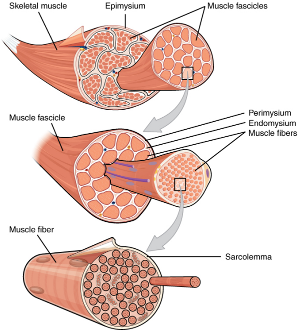
Very complex structure of fibers bundled together

# Musculoskeletal modelling

## Muscles




There are three types of muscles:

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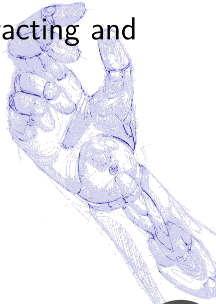
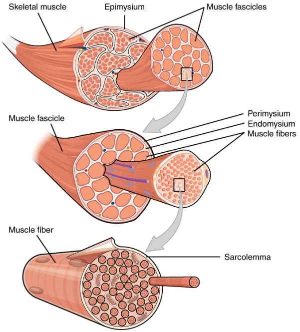
Very complex structure of fibers bundled together

They generate force by contracting and relaxing.



# Musculoskeletal modelling

## Muscles




There are three types of muscles:

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Very complex structure of fibers bundled together

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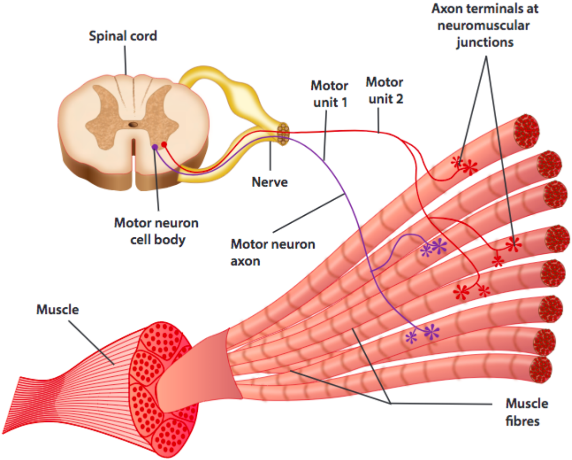
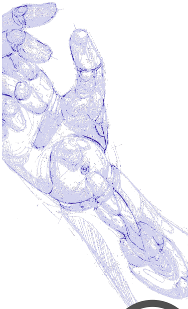
# Musculoskeletal modelling

## Motor units

### Motor units

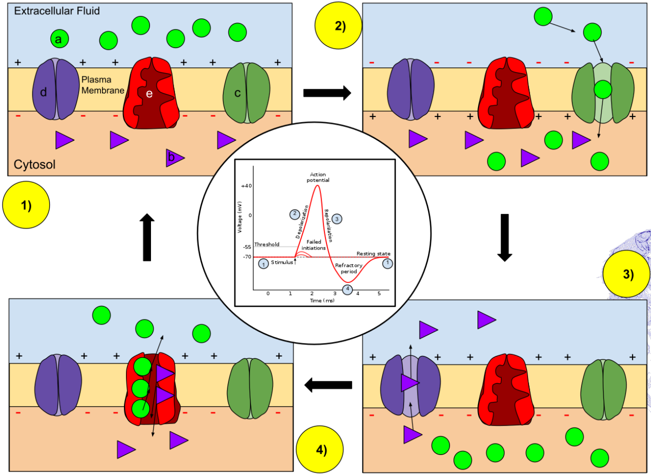
Motor neuron + skeletal muscle.



# Muscle contraction

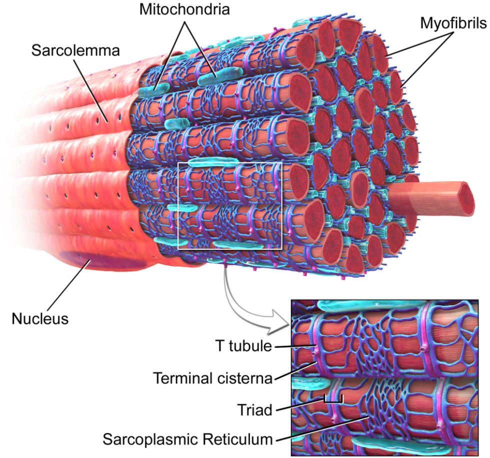
## Action potential




By CThompson02, CC BY-SA 4.0

# Muscle contraction

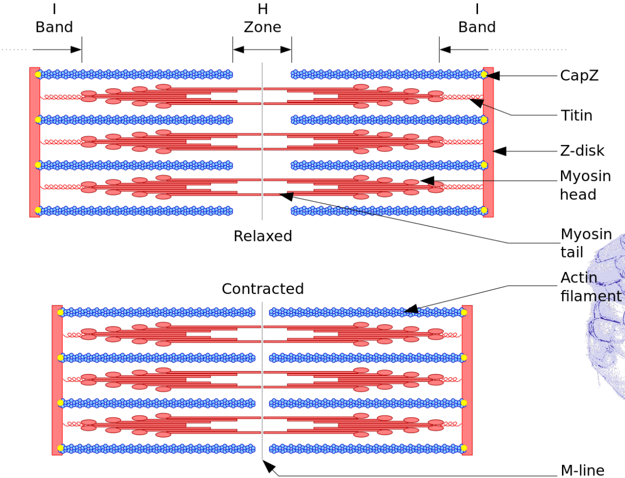
## Muscle fibers




Blausen.com staff (2014)

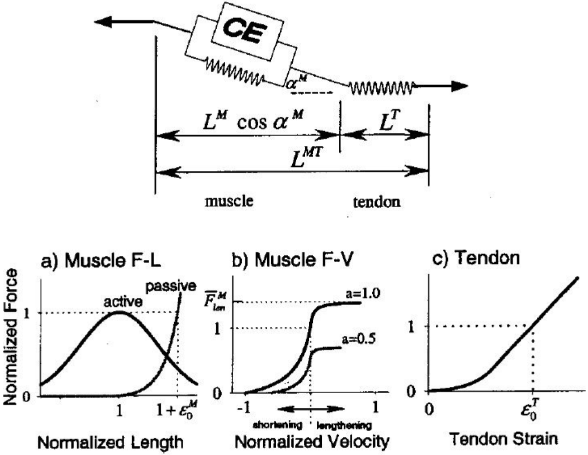
# Muscle contraction

## Sarcomere




Richfield, David (2014)

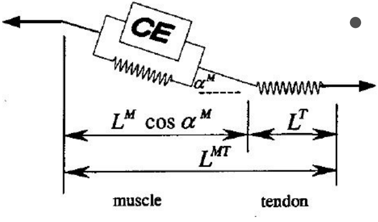
- Maximum isometric force

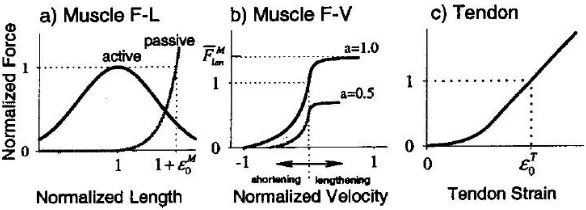



From Thelen (2003)

Muscle models

- Maximum isometric force
- Optimal muscle fiber length

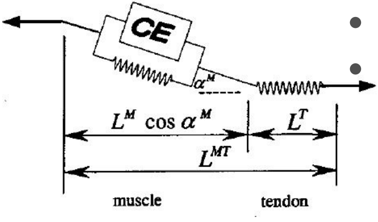


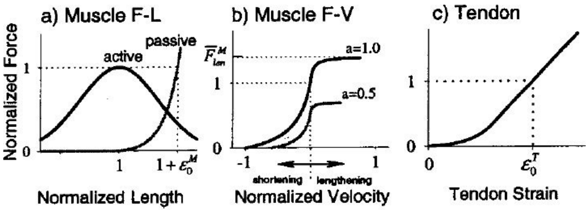

From Thelen (2003)

# Muscluloskeltal modelling

## Muscle models



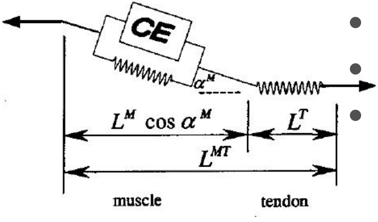

- Maximum isometric force
- Optimal muscle fiber length
- Tendon slack length



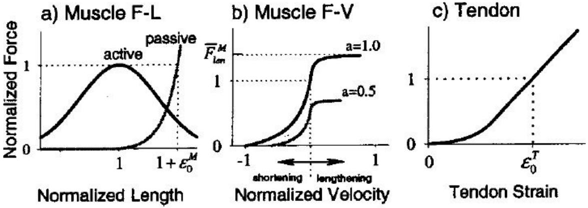

From Thelen (2003)

# Muscluloskeltal modelling

## Muscle models



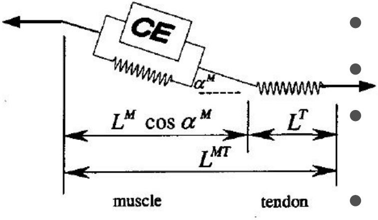

- Maximum isometric force
- Optimal muscle fiber length
- Tendon slack length
- Maximum contraction velocity



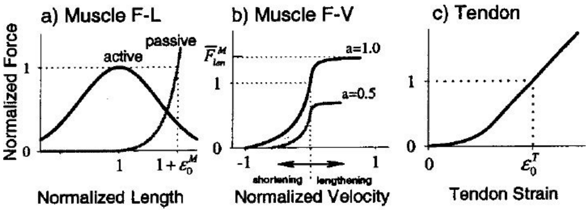

From Thelen (2003)

# Muscluloskeltal modelling

## Muscle models




- Maximum isometric force
- Optimal muscle fiber length
- Tendon slack length
- Maximum contraction velocity
- Pennation angle



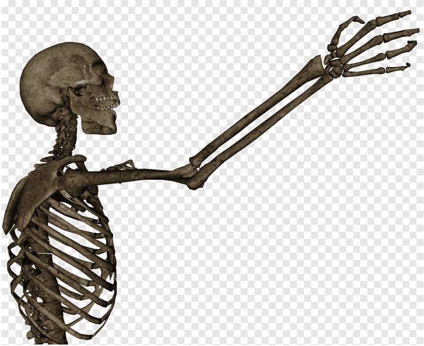

From Thelen (2003)

# Forward kinematics

# Human motion

**Pose**

Description of position and orientation of segments, with respect to a reference frame



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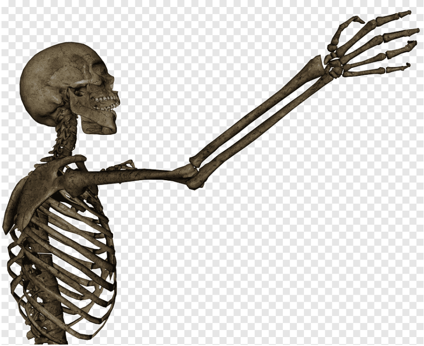

We use coordinate frames

# Forward kinematics

## Definition

The Forward kinematics (FK) is a mathematical tool that allows us to calculate the position and orientation (pose) of a body's point of interest if we know the state of the joints and the lengths of the links.

# Forward kinematics

## Definition

The Forward kinematics (FK) is a mathematical tool that allows us to calculate the position and orientation (pose) of a body's point of interest if we know the state of the joints and the lengths of the links.

**In simple words**

How do I calculate the pose of the human arm if I know the joint angles?

# Forward kinematics

## Definition

We describe the pose of the end-effector using a 4x4 transformation matrix (contains information about position and orientation).

$$T = \left[\begin{array}{ccc|c} 3 & \times & 3 & 3\times 1 \\ \hline 1 & \times & 3 & 1\times 1 \end{array}\right] = \left[\begin{array}{ccc|c} & rotation & & \begin{array}{c} trans- \\ la- \\ tion \end{array} \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \tag{1}$$

# Forward kinematics

## Calculation

To define the FK we perform the following steps:

# Forward kinematics

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- We identify the links and joints of the arm.

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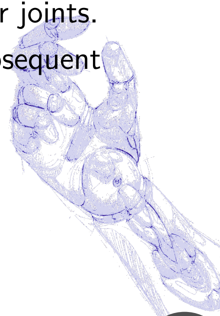
- We attach a coordinate frame on each link at their joints.

# Forward kinematics

## Calculation

To define the FK we perform the following steps:

- We identify the links and joints of the arm.
- We attach a fixed coordinate frame in a convenient location.
- We attach a coordinate frame on each link at their joints.
- We calculate the transformation between each subsequent coordinate frame.



# Forward kinematics

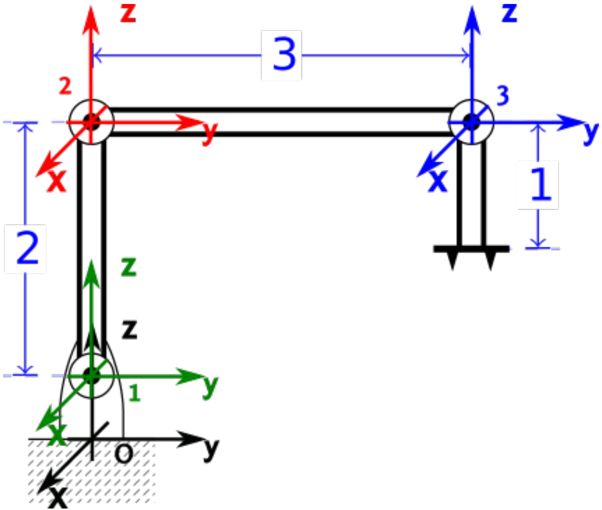
## Calculation

To define the FK we perform the following steps:

- We identify the links and joints of the arm.
- We attach a fixed coordinate frame in a convenient location.
- We attach a coordinate frame on each link at their joints.
- We calculate the transformation between each subsequent coordinate frame.
- We combine the transformations to calculate the overall transformation from base to end-effector.

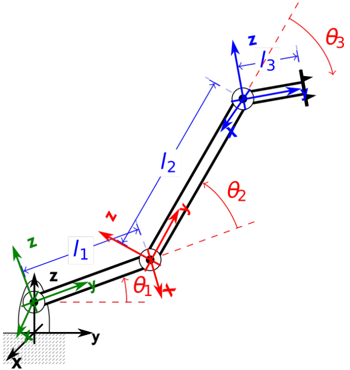
# Forward kinematics

## Calculation




# Forward kinematics

## Dynamic calculation




$$R_3^4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{1,2,3} & -s_{1,2,3} & l_3 c_{1,2,3} + l_2 c_{1,2} + l_1 c_1 \\ 0 & s_{1,2,3} & c_{1,2,3} & l_3 s_{1,2,3} + l_2 s_{1,2} + l_1 s_1 + 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# Inverse kinematics

# Inverse and Forward kinematics

## What is the difference?

**Forward kinematics**

I want to know where will my end-effector be, if I give specific coordinates (values) to each joint

# Inverse and Forward kinematics

## What is the difference?

### Forward kinematics

I want to know where will my end-effector be, if I give specific coordinates (values) to each joint

### Inverse geometric model

I want to know what should the joint coordinates (values) be in order for my end-effector to reach a specific pose

# Inverse and Forward kinematics

## What is the difference?

The inverse model is usually more useful.

# Inverse and Forward kinematics

## What is the difference?

The inverse model is usually more useful.

Can you imagine why?

# Inverse and Forward kinematics

## What is the difference?

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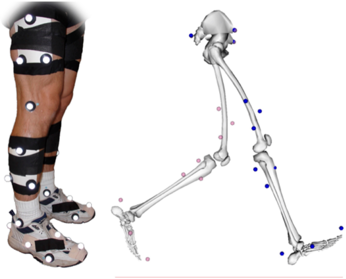

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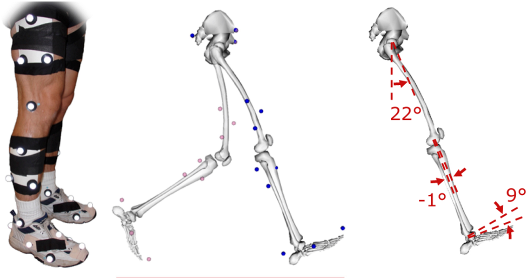


# Inverse and Forward kinematics

## What is the difference?

The inverse model is usually more useful.

But it is also most difficult to derive and we need the forward kinematics to derive it.

# Inverse and Forward kinematics

## What is the difference?

The inverse model is usually more useful.

But it is also most difficult to derive and we need the forward kinematics to derive it.

Can you imagine why?

# Inverse kinematics model

## Derivation

$$\begin{bmatrix} c_{1,2} & -s_{1,2} & 0 & l_2 c_{1,2} + l_1 c_1 \\ s_{1,2} & c_{1,2} & 0 & l_2 s_{1,2} + l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} X_X & Y_X & Z_X & P_x \\ X_Y & Y_Y & Z_Y & P_y \\ X_Z & Y_Z & Z_Z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# Inverse kinematics model

## Derivation

$$\begin{bmatrix} c_{1,2} & -s_{1,2} & 0 & l_2 c_{1,2} + l_1 c_1 \\ s_{1,2} & c_{1,2} & 0 & l_2 s_{1,2} + l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} X_X & Y_X & Z_X & P_x \\ X_Y & Y_Y & Z_Y & P_y \\ X_Z & Y_Z & Z_Z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$cos(q_1 + q_2) = X_x = Y_y$$

$$sin(q_1 + q_2) = X_y = -Y_x$$

$$l_2 cos(q_1 + q_2) + l_1 cos q_1 = P_x$$

$$l_2 sin(q_1 + q_2) + l_1 sin q_1 = P_y$$

$$0 = P_z$$

# Inverse kinematics model

## Derivation

$$\begin{bmatrix} c_{1,2} & -s_{1,2} & 0 & l_2 c_{1,2} + l_1 c_1 \\ s_{1,2} & c_{1,2} & 0 & l_2 s_{1,2} + l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} X_X & Y_X & Z_X & P_x \\ X_Y & Y_Y & Z_Y & P_y \\ X_Z & Y_Z & Z_Z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$cos(q_1 + q_2) = X_x = Y_y$$

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$$l_2 cos(q_1 + q_2) + l_1 cos q_1 = P_x$$

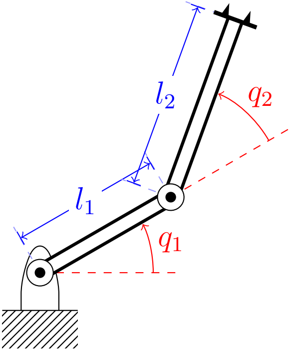
$$l_2 sin(q_1 + q_2) + l_1 sin q_1 = P_y$$

$$0 = P_z$$

How do we solve this?

# Inverse kinematics model

## Examples




$$R(q_1, q_2) = \begin{bmatrix} c_{1,2} & -s_{1,2} & 0 & l_2 c_{1,2} + l_1 c_1 \\ s_{1,2} & c_{1,2} & 0 & l_2 s_{1,2} + l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# Inverse kinematics model

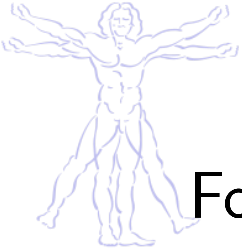
## Analytical solution

$$q_2 = cos^{-1}\frac{x^2 + y^2 - l_1^2 - l_2^2}{2l_1l_2}$$

$$q_1 = atan2(x, y) - \beta = atan2(y, x) - atan2(k_2, k_1)$$

Where:

$$k_1 = l_1 + l_2cos(q_2)$$
$$k_2 = l_2sin(q_2)$$



# Forward dynamics

# Dynamic modeling

## What is it all about?

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**Dynamics (Kinetics)**:

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# Dynamic modeling

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**Kinematics**: description of motion of bodies or system of bodies

**Dynamics (Kinetics)**: description of the causes resulting in those motions (i.e. forces and torques)

# Dynamic modeling

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### Dynamic model

A set of equations that gives us the relationship between input joint forces/torques and resulting joint accelerations.

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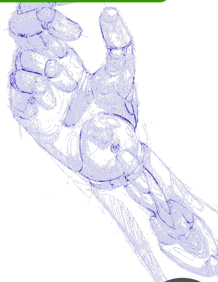
Why is this useful?



# Lagrangian mechanics

## A more sophisticated formulation of mechanics

Lagrange defined a basic quantity for any system of bodies as the difference between its kinetic and potential energy.

$$L = K - P$$

We call this quantity the Lagrangian of the system.

# Definitions

## Potential energy

### Total potential energy

The total potential energy of a mechanism is the sum of the potential energy of its parts

# Definitions

## Kinetic energy

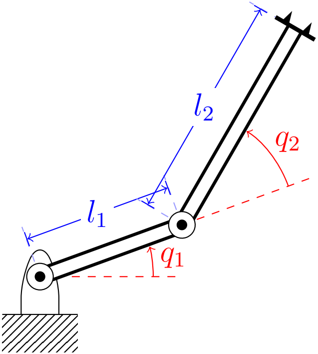
### Total kinetic energy

The total kinetic energy of an object is the sum of its linear and angular kinetic energy.

$$K_{total} = K_{linear} + K_{angular} = \frac{1}{2}(mu^2 + I\omega^2)$$

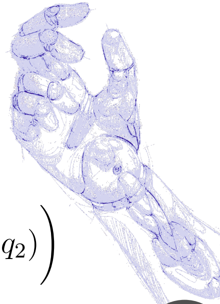
# Lagrangian of a mechanism

## Potential energy




The total potential energy is:

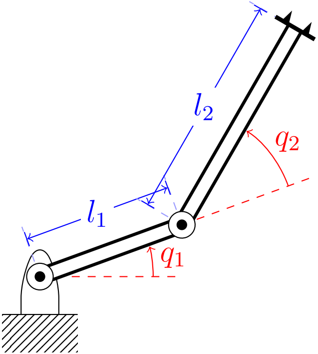
$$P(q,\dot{q}) = m_1 g \frac{l_1}{2} sinq_1 + m_2 g \left( l_1 sinq_1 + \frac{l_2}{2} sin(q_1+q_2) \right)$$



# Lagrangian of a mechanism

## Kinetic energy

The total Kinetic energy of the mechanism is:




$$K(q,\dot{q}) = \frac{1}{2}\dot{q}^T \sum_{i=1}^{n} \left[ J_{vi}^T m_i J_{vi} + J_{\omega i}^T R_i I_i R_i^T J_{\omega i} \right] \dot{q} = \frac{1}{2}\dot{q}^T D(q)\dot{q}$$

# Lagrangian of a mechanism

## Let's plug it all together

The equation of motion is:

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# Lagrangian of a mechanism

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$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k} - \frac{\partial L}{\partial q_k} = \tau_k$$

# Lagrangian of a mechanism

## Condensed form

We can write this equation in a more general form:

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau$$

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Finally, the term $g$ contains the dependence of the potential energy from the position of the mechanism.

# Dynamic model

## Torques

How do we calculate torques?

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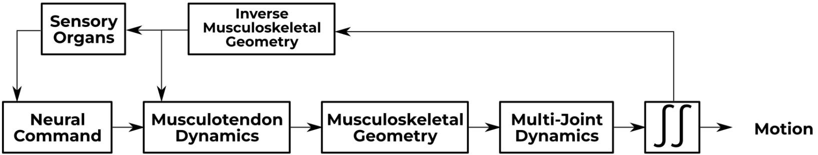


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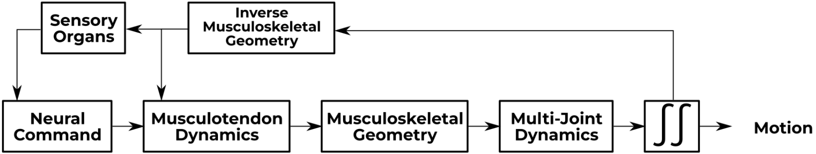


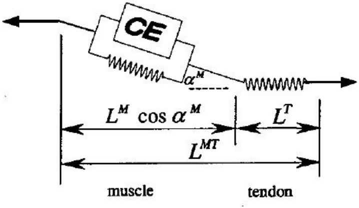


$$f_{iso}(\alpha(t) f_{AL}(l^M) f_v(i^M) + f_{PL}(l^M)) cosa - f_{iso} f_{SE}(l^T) = 0$$

# Dynamic model

## Forward dynamics

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

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$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

$$\ddot{q} = D(q)^{-1}[\tau - C(q,\dot{q})\dot{q} - g(q)]$$

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$$\ddot{q} = D(q)^{-1}[\tau - C(q,\dot{q})\dot{q} - g(q)]$$

$$\ddot{q} = D(q)^{-1}[\tau(\alpha, l, \dot{l}) - C(q,\dot{q})\dot{q} - g(q)]$$

# Inverse dynamics

# Dynamic model

## Inverse dynamics

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

# Coming up next

# Questions?