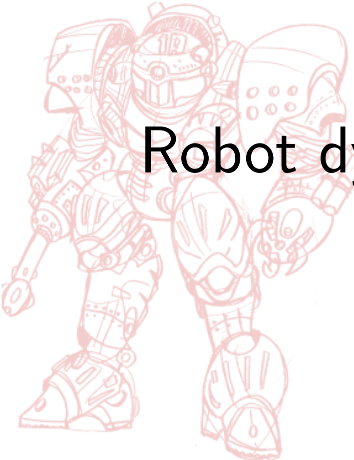


Robot dynamic modeling

TECHNICAL UNIVERSITY OF CLUJ-NAPOCA ROMANIA

October 14, 2018

Agenda

- Background
- Dynamics
- The Lagrangian
- Dynamic and Kinetic energy
- Inertia and moments of intertia

What did we do last week?

Recap

We defined a matrix that we called the Jacobian, that maps joint velocities to link velocities.

$$u = J\dot{q}$$

What did we do last week?

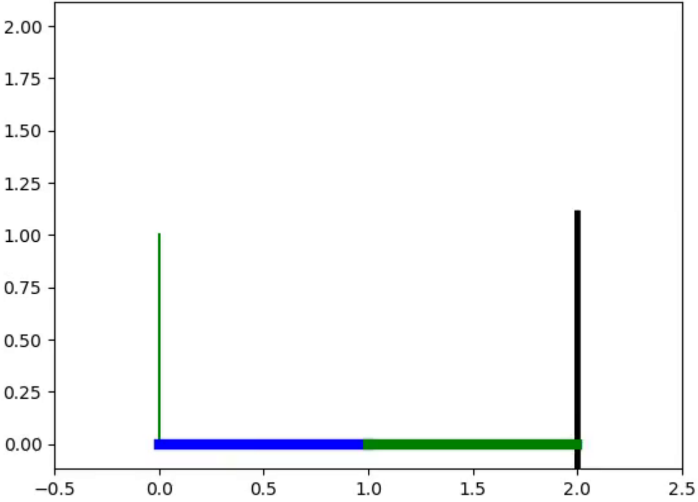
Recap

We split this matrix into two parts, one for the angular and one for the linear velocity of the links.

$$\dot{x} = J_u \dot{q}$$
$$\omega = J_{omega} \dot{q}$$

What did we do last week?

Recap



Dynamic modeling

What is it all about?

Kinematics:

Dynamics (Kinetics):

Dynamic modeling

What is it all about?

Kinematics: description of motion of bodies or system of bodies

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Dynamic modeling

What is it all about?

Kinematics: description of motion of bodies or system of bodies

Dynamics (Kinetics): desctiption of the causes resulting in those motions (i.e. forces and torques)

Dynamic modeling

What is it all about?

Dynamic model

A set of equations that gives us the relationship between input joint forces/torques and resulting joint accelerations.

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Why is this useful?

Dynamic modeling

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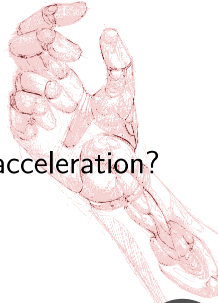
Dynamic model

A set of equations that gives us the relationship between input joint forces/torques and resulting joint accelerations.

Why is this useful?

Do you know of any equation that relates force with acceleration?



Dynamic modeling

Newton's equations

$$F = m\ddot{x}$$

This is the famous equation derived from Newton's second law, which relates force and acceleration.

Dynamic modeling

Newton's equations

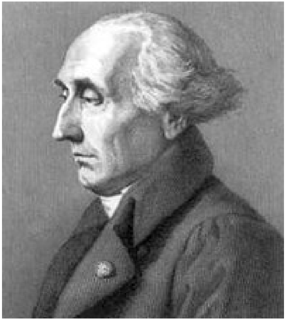
$$F = m\ddot{x}$$

This is the famous equation derived from Newton's second law, which relates force and acceleration.

We won't be using it though due to the complex equetions it will result when modeling kinematic chains.

Dynamic modeling

Lagrange-Euler formulation of mechanics



Between 1772 and 1788, Lagrange formulated mechanics in a more general way, more suitable for robotics later on.

Lagrangian mechanics

A more sophisticated formulation of mechanics

Lagrange defined a basic quatity for any system of bodies as the difference between its kinetic and potential energy.

$$L = K - P$$

We call this quantity the Lagrangian of the system.

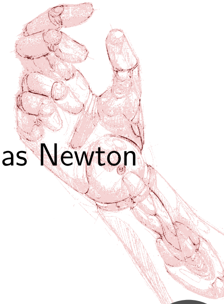
Lagrangian mechanics

A more sophisticated formulation of mechanics

Using this quantity, we can describe the evolution of any system of bodies under the influence of a set of external forces using the following equation:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{y}} - \frac{\partial L}{\partial y} = \tau$$

Let's see if we can show it gives us the same results as Newton



Definitions

Potential energy

Potential energy

The energy possessed by an object because of its position relative to other objects, stresses within itself, its electric charge, or other factors.

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The energy possessed by an object because of its position relative to other objects, stresses within itself, its electric charge, or other factors.

In robotics, we deal with rigid objects, electrically neutral. What are the sources of potential energy?

Definitions

Potential energy

The most common source of potential energy in robotics is the gravitational field of Earth.

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Where m is the mass of the object, g is the gravitational constant, and h is the height of the object.

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But height from where? What is the reference?

Definitions

Potential energy

Reference for potential

Potential is only important when considering the difference of potential. Therefore, the reference is not important, as long as it does not change over time, and we use the same one for all the objects.

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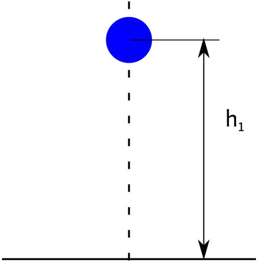

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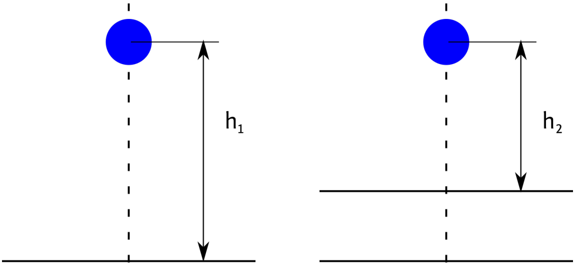

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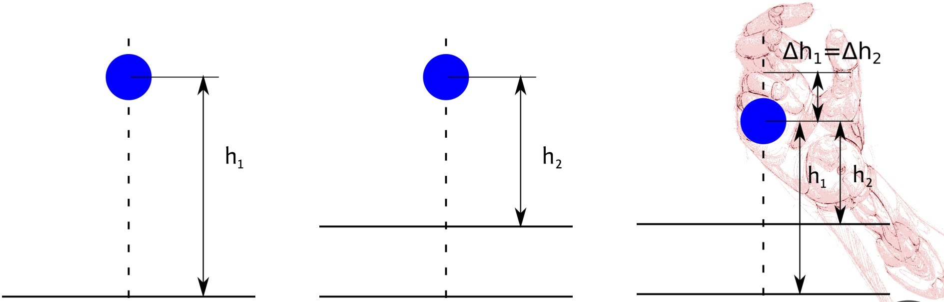

Definitions

Kinetic energy

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The energy of an object that it possesses due to its motion.

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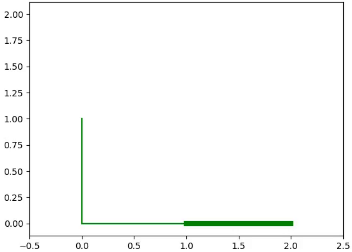
What properties are influencing the kinetic energy of an object.

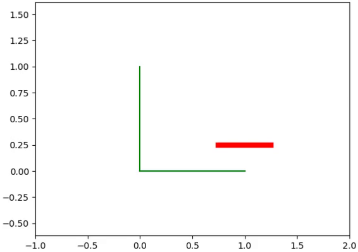
Definitions

Kinetic energy



Kinetic energy due to

linear velocity

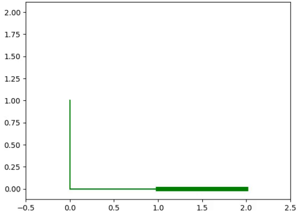


Kinetic energy due to

angular velocity

Definitions

Kinetic energy



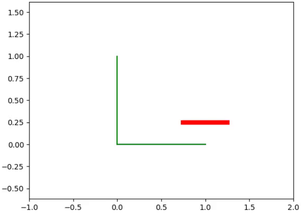
The equation of kinetic energy due to linear velocity is:

$$K_{linear} = \frac{1}{2}mu^2$$

Where m is the mass of the object and u is the magnitude of its velocity (i.e. regardless of direction).

Definitions

Kinetic energy



The equation of kinetic energy due to angular velocity is:

$$K_{angular} = \frac{1}{2} I \omega^2$$

Where I is the moment of inertia of the object, and ω is its angular velocity.

Definitions

Kinetic energy

Total kinetic energy

The total kinetic energy of an object is the sum of its linear and angular kinetic energy.

$$K_{total} = K_{linear} + K_{angular} = \frac{1}{2}(mu^2 + I\omega^2)$$

Definitions

Moment of inertia

Moment of inertia

A tensor that determines the torque needed for a desired angular acceleration about a rotational axis.

The moment of inertia is the equivalent of mass, but for rotational movements.

Definitions

Moment of inertia

The moment of inertia shows us how 'difficult' is it to rotate an object around an arbitrary axis. It is related with how the mass of the object is distributed in space.

$$I = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}$$

This 'difficulty' might be different for the same object, but different axes.

Definitions

Moment of inertia

How do we calculate it? (for a system of bodies)

$$I_{xx} = \sum_{k=1}^{N} m_k (y_k^2 + z_k^2)$$

$$I_{yy} = \sum_{k=1}^{N} m_k (x_k^2 + z_k^2)$$

$$I_{zz} = \sum_{k=1}^{N} m_k (x_k^2 + y_k^2)$$

$$I_{xy} = I_{yx} = -\sum_{k=1}^{N} m_k x_k y_k$$

$$I_{xz} = I_{zx} = -\sum_{k=1}^{N} m_k x_k z_k$$

$$I_{yz} = I_{zy} = -\sum_{k=1}^{N} m_k y_k z_k$$

Definitions

Moment of inertia

How do we calculate it? (for a continuous object)

$$I_{xx} = \iiint (y^2 + z^2)\rho(x,y,z)dV \qquad I_{xy} = I_{yx} = -\iiint xy\rho(x,y,z)dV$$

$$I_{yy} = \iiint (x^2 + z^2)\rho(x,y,z)dV \qquad I_{xz} = I_{zx} = -\iiint xz\rho(x,y,z)dV$$

$$I_{zz} = \iiint (x^2 + y^2)\rho(x,y,z)dV \qquad I_{yz} = I_{zy} = -\iiint yz\rho(x,y,z)dV$$

Definitions

Moment of inertia

Seems difficult?

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For some simple objects, we have analytical solutions for the moment of inertia tensor.

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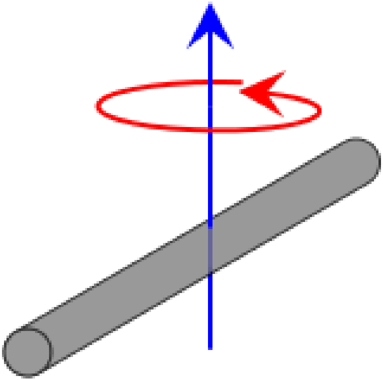

$$I = \begin{bmatrix} \frac{1}{12}ml^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{12}ml^2 \end{bmatrix}$$

Definitions

Kinetic energy

Let's have a look at the angular kinetic energy again. We saw that:

$$K_{angular} = \frac{1}{2} I \omega^2$$

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Kinetic energy

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$$K_{angular} = \frac{1}{2} I \omega^2$$

But if I, is a tensor and ω a scalar, then the kinetic energy will be a tensor as well.

Definitions

Kinetic energy

But Kinetic energy is a scalar, and the angular velocity is a vector.

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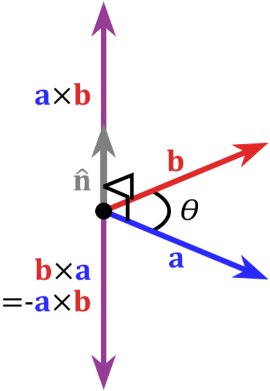
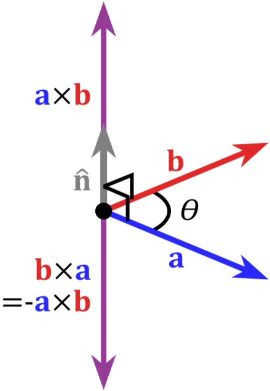


Definitions

Kinetic energy

But Kinetic energy is a scalar, and the angular velocity is a vector. Therefore, we can calculate the angular kinetic energy using the vectorial equation:



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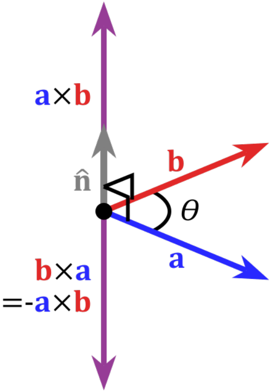


$$K_{angular} = \frac{1}{2}\omega^T I \omega$$

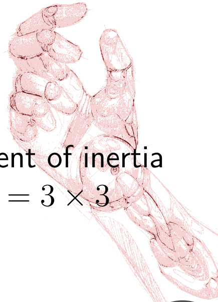
Definitions

Bringing it all together

We define the Lagrangian as the difference between Kinetic and Potential energy of our system

$$L = K - P$$

where:



Potential Energy

$$P = mgh$$

Kinetic Energy

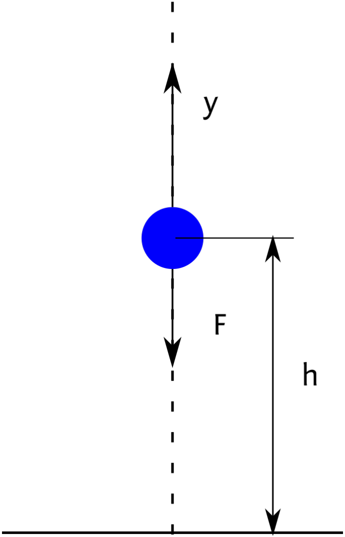
$$K = \frac{1}{2}(mu^2 + \omega^T I \omega)$$

Moment of inertia

$$I = 3 \times 3$$

Lagrangian mechanics

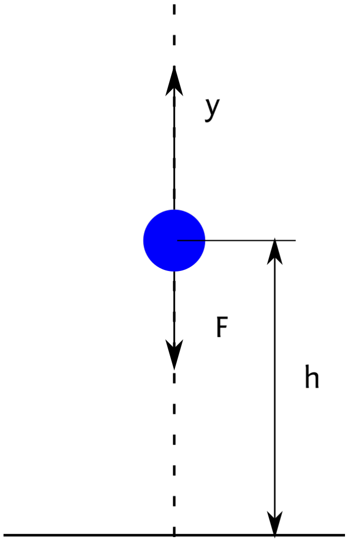
Equivalence with Newton



Let's see if the Lagrangian mechanics are giving us the same results

Lagrangian mechanics

Equivalence with Newton



Potential energy:

$$P = mgy$$

Kinetic energy:

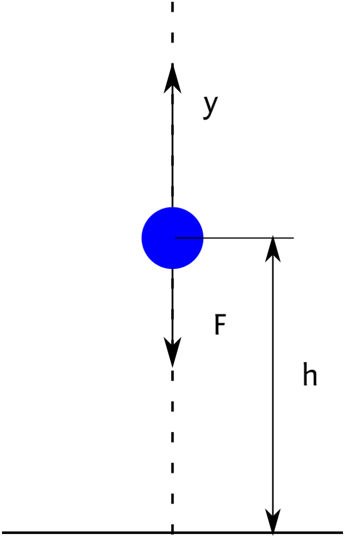
$$K = \frac{1}{2}m\dot{y}^2$$

Therefore:

$$L = \frac{1}{2}m\dot{y}^2 - mgy$$

Lagrangian mechanics

Equivalence with Newton



$$L = \frac{1}{2}m\dot{y}^2 - mgy$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{y}} - \frac{\partial L}{\partial y} = F$$

Lagrangian mechanics

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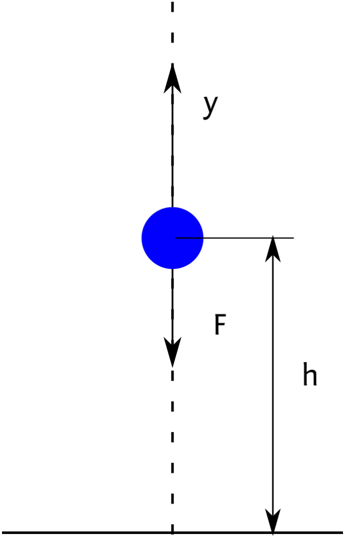

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$$\frac{\partial L}{\partial \dot{y}} = m\dot{y}$$

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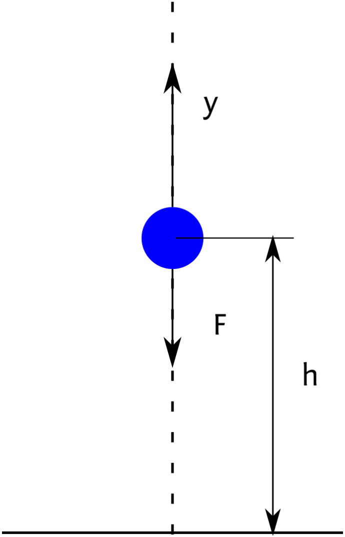

$$L = \tfrac{1}{2} m \dot{y}^2 - mgy$$

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$$\frac{\partial L}{\partial \dot{y}} = m\dot{y}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{y}} = m\ddot{y}$$

Lagrangian mechanics

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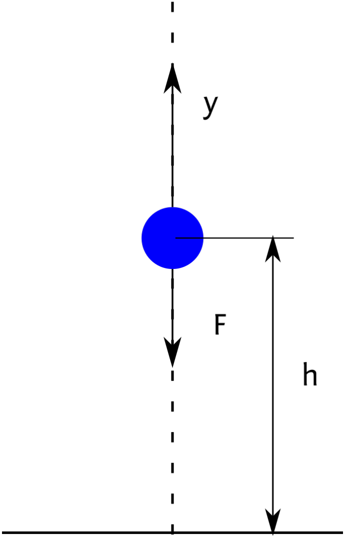

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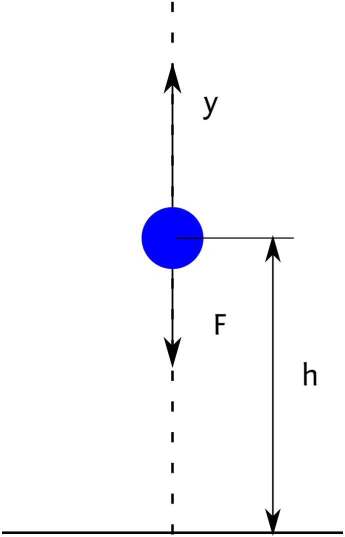
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{y}} = m\ddot{y}$$

$$\frac{\partial L}{\partial y} = mg$$

Lagrangian mechanics

Equivalence with Newton



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Lagrangian mechanics

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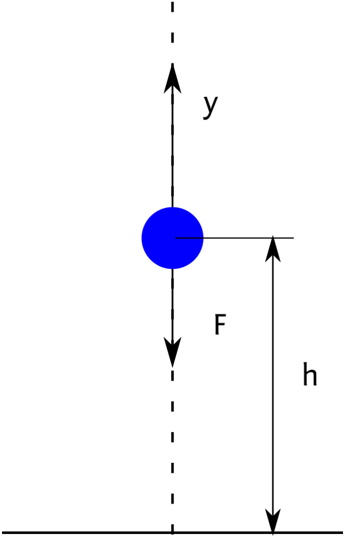

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{y}} - \frac{\partial L}{\partial y} = F$$

$$m\ddot{y} - mg = F$$

Same as Newton's second law!

Questions?